



Note

Covering radii are not matroid invariants

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Abstract

We show by example that the covering radius of a binary linear code is not generally determined by the Tutte polynomial of the matroid. This answers Problem 361 (P.J. Cameron (Ed.), Research problems, Discrete Math. 231 (2001) 469–478).

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The celebrated “Critical Theorem” by Crapo and Rota [2] shows that many detailed properties of a linear code $C \subseteq \mathbb{F}^E$ (over a field \mathbb{F} , with coordinates labeled by the elements of a set E) are determined by the associated vector matroid M_C . Greene [3] further demonstrated that the Tutte polynomial

$$T(M_C; x, y) = \sum_{A \subseteq E} (x-1)^{\rho_{M_C}(E) - \rho_{M_C}(A)} (y-1)^{|A| - \rho_{M_C}(A)}$$

often suffices to determine properties of C . Examples of such properties include the code length, dimension, minimum distance, and the weight enumerator. The purpose of this note is to present general code properties that are *not* determined by the Tutte polynomial of the associated matroid.

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The covering radius $r(C)$ of a code $C \subseteq \mathbb{F}^E$ is the maximal distance from C to any vector of \mathbb{F}^E . Equivalently, it is one less than the cardinality of the weight distribution, a_0, a_1, \dots, a_r , of coset leaders v in the cosets $v + C$. The matrices

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \text{ and } \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

over $\text{GF}(2)$ represent matroids with a common Tutte polynomial given by

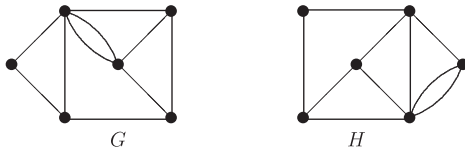
$$y^5 + (x + 5)y^4 + (x^2 + 7x + 12)y^3 + (x^3 + 8x^2 + 22x + 15)y^2 + (2x^4 + 11x^3 + 27x^2 + 27x + 7)y + (x^6 + 5x^5 + 13x^4 + 21x^3 + 19x^2 + 7x),$$

but generate a pair of codes with covering radii 2 and 3, respectively. Thus,

Theorem 1. *The covering radius $r(C)$ of a binary linear code C is not generally determined by the Tutte polynomial $T(M_C; x, y)$.*

Theorem 1 answers in the negative the question posed in [1, Problem 361].

Gray (see [6]) observed that the pair of graphs presented below share a common Tutte polynomial, even though they are not 2-isomorphic.



Let $C(G)$ and $C(H)$ be the bond codes of G and H , i.e. the binary linear codes spanned by the characteristic vectors of the bonds of G and H , respectively. The non-isomorphic matroids $M_{C(G)} = M(G)$ and $M_{C(H)} = M(H)$ have in common their Tutte polynomial. Furthermore, the codes $C(G)$ and $C(H)$ both have covering radius 3. However, the weight distributions of their coset leaders are 1, 9, 20, 2 and 1, 9, 18, 4, respectively. Therefore,

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