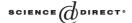


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Discrete Mathematics 292 (2005) 107-117

www.elsevier.com/locate/disc

# Ramsey numbers of stars versus wheels of similar sizes

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Received 19 October 2003; received in revised form 29 November 2004; accepted 16 December 2004

#### Abstract

We study the Ramsey number  $R(W_m, S_n)$  for a star  $S_n$  on n vertices and a wheel  $W_m$  on m+1 vertices. We show that the Ramsey number  $R(W_m, S_n) = 3n-2$  for n=m, m+1, and m+2, where  $m \ge 7$  and odd. In addition, we give the following lower bound for  $R(W_m, S_n)$  where m is even:  $R(W_m, S_n) \ge 2n+1$  for all  $n \ge m \ge 6$ .

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Keywords: Ramsey number; Star; Wheel

#### 1. Introduction

For two graphs G and H, the Ramsey number R(G, H) is the smallest positive integer r such that for every graph F on r vertices, F contains G as a subgraph or the complement of F contains H as a subgraph.

In this paper, we study the Ramsey number  $R(W_m, S_n)$  of wheels versus stars. A wheel  $W_m$  is the graph on m+1 vertices obtained from a cycle  $C_m$  on m vertices by adding one vertex o, called the *hub* of the wheel, and making o adjacent to all vertices of  $C_m$ , called the *rim* of the wheel. A *star*  $S_n$  is the graph on n vertices with one vertex of degree n-1, called the *center*, and n-1 vertices of degree 1.

It was shown in [5] by Surahmat et al. that  $R(W_m, S_n) = 3n - 2$  for  $n \ge 2m - 4$ , where  $m \ge 5$  and odd. It was also shown in [4] that  $R(W_4, S_n) = 2n - 1$  if  $n \ge 3$  and odd,  $R(W_4, S_n) = 2n + 1$  if  $n \ge 4$  and even, and  $R(W_5, S_n) = 3n - 2$  for each  $n \ge 3$ . Baskoro et al. have also shown

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in [1] that  $R(W_4, T_n) = 2n - 1$  for  $n \ge 4$  and  $R(W_5, T_n) = 3n - 2$  for  $n \ge 3$  for any tree  $T_n$  on n vertices that is not a star.

In this paper we prove that  $R(W_m, S_n) = 3n - 2$  for n = m, m + 1, and m + 2, where  $m \ge 7$  and odd. In particular, this completes the calculation that  $R(W_7, S_n) = 3n - 2$  for each  $n \ge 7$ . In addition, we give the following lower bound:  $R(W_m, S_n) \ge 2n + 1$  for all  $n \ge m \ge 6$  and m even.

#### 2. Background

Let G be a graph with vertex set V(G) and edge set E(G). For  $v \in V(G)$  and  $B \subset V(G)$ , define  $N_B(v) = \{y \in B : vy \in E(G)\}$ . Define the *degree of v with respect to B* to be  $|N_B(v)|$  and denote it by  $\mathcal{D}_B(v)$ . If B consists of the entire vertex set of the graph G (i.e. B = V(G)), we use the conventional  $d_G(v)$  instead of  $\mathcal{D}_{V(G)}(v)$ .

Let  $\overline{G}$  denote the complement of G, i.e. the graph obtained from the complete graph on the vertices of G by deleting the edges of G.

Chvátal and Harary [2] established the following lower bound for Ramsey numbers:

$$R(G, H) \ge (\mathcal{X}(G) - 1) \cdot (c(H) - 1) + 1$$
,

where  $\mathcal{X}(G)$  is the chromatic number of G and c(H) is the number of vertices in the largest connected component of H.

**Corollary 1.** 
$$R(W_{2k+1}, S_n) \ge 3n - 2$$
 for  $n \ge 2k + 1$ .

The inequality follows directly from the Chvátal and Harary bound and the facts that  $\mathcal{X}(W_{2k+1}) = 4$  and  $c(S_n) = n$ .

Corollary 2. 
$$R(W_{2k}, S_n) \ge 2n - 1$$
 for  $n \ge 2k$ .

The inequality here follows directly from the Chvátal and Harary bound and the facts that  $\mathcal{X}(W_{2k}) = 3$  and  $c(S_n) = n$ .

The following well-known theorem [3] is useful throughout the paper:

**Dirac's Theorem.** Every graph with  $n \ge 3$  vertices and minimum degree at least n/2 has a Hamiltonian cycle.

3. 
$$R(W_n, S_n) = 3n - 2$$
 when *n* is odd

**Theorem 3.** 
$$R(W_{2k+1}, S_{2k+1}) = 6k + 1$$
 for  $k \ge 3$ .

**Proof.** Corollary 1 yields  $R(W_{2k+1}, S_{2k+1}) \ge 3 \cdot (2k+1) - 2 = 6k+1$ . Therefore, it suffices to prove that  $R(W_{2k+1}, S_{2k+1}) \le 6k+1$ .

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