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Ramsey numbers of stars versus wheels of similar sizes

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Abstract

We study the Ramsey number $R(W_m, S_n)$ for a star S_n on n vertices and a wheel W_m on $m+1$ vertices. We show that the Ramsey number $R(W_m, S_n) = 3n-2$ for $n=m$, $m+1$, and $m+2$, where $m \geqslant 7$ and odd. In addition, we give the following lower bound for $R(W_m, S_n)$ where m is even: $R(W_m, S_n) \geqslant 2n+1$ for all $n \geqslant m \geqslant 6$.

Keywords: Ramsey number; Star; Wheel

1. Introduction

For two graphs G and H, the *Ramsey number* $R(G, H)$ is the smallest positive integer r such that for every graph F on r vertices, F contains G as a subgraph or the complement of F contains H as a subgraph.

In this paper, we study the Ramsey number $R(W_m, S_n)$ of wheels versus stars. A *wheel* W_m is the graph on $m+1$ vertices obtained from a cycle C_m on m vertices by adding one vertex o, called the *hub* of the wheel, and making o adjacent to all vertices of C_m, called the *rim* of the wheel. A *star* S_n is the graph on n vertices with one vertex of degree $n-1$, called the *center*, and $n-1$ vertices of degree 1.

It was shown in [5] by Surahmat et al. that $R(W_m, S_n)=3n-2$ for $n \geqslant 2m-4$, where $m \geqslant 5$ and odd. It was also shown in [4] that $R(W_4, S_n)=2n-1$ if $n \geqslant 3$ and odd, $R(W_4, S_n)=2n+1$ if $n \geqslant 4$ and even, and $R(W_5, S_n) = 3n-2$ for each $n \geqslant 3$. Baskoro et al. have also shown

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in [1] that $R(W_4, T_n) = 2n - 1$ for $n \geqslant 4$ and $R(W_5, T_n) = 3n - 2$ for $n \geqslant 3$ for any tree T_n on n vertices that is not a star.

In this paper we prove that $R(W_m, S_n) = 3n - 2$ for $n = m, m + 1$, and $m + 2$, where $m \geqslant 7$ and odd. In particular, this completes the calculation that $R(W_7, S_n) = 3n - 2$ for each $n \geqslant 7$. In addition, we give the following lower bound: $R(W_m, S_n) \geqslant 2n + 1$ for all $n \geqslant m \geqslant 6$ and m even.

2. Background

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. For $v \in V(G)$ and $B \subset V(G)$, define $N_B(v) = \{y \in B : vy \in E(G)\}$. Define the *degree of* v *with respect to* B to be $|N_B(v)|$ and denote it by $\mathscr{D}_B(v)$. If B consists of the entire vertex set of the graph G (i.e. $B = V(G)$), we use the conventional $d_G(v)$ instead of $\mathscr{D}_{V(G)}(v)$.

Let $\overline{G}$ denote the complement of G, i.e. the graph obtained from the complete graph on the vertices of G by deleting the edges of G.

Chvátal and Harary [2] established the following lower bound for Ramsey numbers:

$$R(G, H) \geqslant (\mathscr{X}(G) - 1) \cdot (c(H) - 1) + 1,$$

where $\mathscr{X}(G)$ is the chromatic number of G and $c(H)$ is the number of vertices in the largest connected component of H.

Corollary 1. $R(W_{2k+1}, S_n) \geqslant 3n - 2$ *for* $n \geqslant 2k + 1$.

The inequality follows directly from the Chvátal and Harary bound and the facts that $\mathscr{X}(W_{2k+1}) = 4$ and $c(S_n) = n$.

Corollary 2. $R(W_{2k}, S_n) \geqslant 2n - 1$ *for* $n \geqslant 2k$.

The inequality here follows directly from the Chvátal and Harary bound and the facts that $\mathscr{X}(W_{2k}) = 3$ and $c(S_n) = n$.

The following well-known theorem [3] is useful throughout the paper:

Dirac's Theorem. *Every graph with* $n \geqslant 3$ *vertices and minimum degree at least* $n/2$ *has a Hamiltonian cycle.*

3. $R(W_n, S_n) = 3n - 2$ when n is odd

Theorem 3. $R(W_{2k+1}, S_{2k+1}) = 6k + 1$ *for* $k \geqslant 3$.

Proof. Corollary 1 yields $R(W_{2k+1}, S_{2k+1}) \geqslant 3 \cdot (2k + 1) - 2 = 6k + 1$. Therefore, it suffices to prove that $R(W_{2k+1}, S_{2k+1}) \leqslant 6k + 1$.