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Circumferences and Minimum Degrees in 3-Connected Claw-Free Graphs

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Abstract

In this paper, we prove that every 3-connected claw-free graph on n vertices contain a cycle of length at least $\min\{n, 6\delta - 17\}$, hereby improving several known results.

Keywords: Circumference, longest cycle, claw-free graph.

1 Introduction

We only consider loopless finite simple graphs, and use [1] for terminology and notations not defined here. A graph G is *eulerian* if G is connected and

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every vertex of G is of degree even. A circuit C of a graph G is a connected eulerian subgraph of G. A cycle is a connected circuit with all vertices of degree 2. Let C be a circuit of a graph G. The *minimum degree* and the *edge independence number* of G are denoted by $\delta(G)$ (or δ) and $\alpha'(G)$, respectively. An edge $e = uv$ is called a *pendant edge* if either $d_G(u) = 1$ or $d_G(v) = 1$. A subgraph H of G (denoted by $H \subseteq G$) is *dominating* if $G - V(H)$ is edgeless. For $x \in V(G)$, let $N_H(x) = \{v \in V(H)\colon vx \in E(G)\}$ and $d_H(x) = |N_H(x)|$. If $S \subseteq V(G)$, $G[S]$ is the subgraph induced in G by S. A vertex $v \in G$ is called a *locally connected vertex* if $G[N_G(v)]$ is connected. For $A, B \subseteq V(G)$ with $A \cap B = \emptyset$, let $N_H(A) = \cup_{v\in A} N_H(v)$, $E_G[A, B] = \{uv \in E(G) \mid u \in A, v \in B\}$, and $G - A = G[V(G) - A]$. When $A = \{v\}$, we use $G - v$ for $G - \{v\}$. If $H \subseteq G$, then for an edge subset $X \subseteq E(G) - E(H)$, we write $H + X$ for $G[E(H) \cup X]$. For an integer $i \geq 1$, define $D_i(G) = \{v \in V(G) | d_G(v) = i\}$.

A graph H is claw-free if it does not contain $K_{1,3}$ as an induced subgraph. The *line graph* of a graph G, denote by $L(G)$, has $E(G)$ as its vertex set, where two vertices in $L(G)$ are adjacent if and only if the corresponding edges in G are adjacent. Obviously, a line graph is claw-free. Let H be the line graph $L(G)$ of a graph G. Then $|V(H)| = |E(G)|$ and $\delta(H) = \min\{d_G(x) + d_G(y) - 2\colon xy \in E(G)\}$. If $L(G)$ is k-connected, then G is *essentially k-edge-connected*, which means that the only edge-cut sets of G having less than k edges are the sets of edges incident with some vertex of G. Harary and Nash-Williams [6] showed that there is a closed relationship on hamiltonian cycles between a graph and its line graph.

Theorem 1.1 (Harary and Nash-Williams [6]). *The line graph $L(G)$ of a graph G is hamiltonian if and only if G has a dominating eulerian subgraph.*

Ryjáček [15] defined the *closure* $cl(H)$ of a claw-free graph H to be one obtained by recursively adding edges to join two nonadjacent vertices in the neighborhood of any locally connected vertex of H, as long as this is possible, and proved the following result.

Theorem 1.2 (Ryjáček [15]). *Let H be a claw-free graph and $cl(H)$ its closure. Then*
(i) $cl(H)$ is well-defined, and $\kappa(cl(H)) \geq \kappa(H)$,
(ii) there is triangle-free graph G such that $cl(H) = L(G)$,
(iii) both graphs H and $cl(H)$ have the same circumference.

Many works have been done to give sufficient conditions for a claw-free graph H to be hamiltonian in terms of its minimum degree $\delta(H)$. These conditions