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Perfect codes in direct products of cycles

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Abstract

Let $G = \times_{i=1}^{n} C_{\ell_i}$ be the direct product of cycles. It is proved that for any $r \geq 1$, and any $n \geq 2$, each connected component of G contains an r-perfect code provided that each ℓ_i is a multiple of $r^n + (r+1)^n$. On the other hand, if a code of G contains a given vertex and its canonical local vertices, then any ℓ_i is a multiple of $r^n + (r+1)^n$. It is also proved that an r-perfect code ($r \geq 2$) of G is uniquely determined by n vertices and it is conjectured that for $r \geq 2$ no other codes in G exist than the constructed ones.

Keywords: Error-correcting codes, Direct product of graphs, Perfect codes, cycles

1 introduction

The study of codes in graphs presents a wide generalization of the problem of the existence of (classical) error-correcting codes. In general, for a given graph G we search for a subset X of its vertices such that the r-balls of vertices from X form a partition of the vertex set of G. Hamming codes and Lee codes correspond to codes in the Cartesian product of complete graphs and cycles, respectively.

The study of codes in graphs was initiated by Biggs [1] who rightly noticed that the class of all graphs is a too general setting and hence restricted himself to

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distance-transitive graphs. Kratochvíl continued the study of (perfect) codes in graphs, see [7] and references therein. While the Cartesian product of graphs covers some classical error-correcting codes, the direct product of graphs is another interesting graph product with respect to codes in graphs. This graph product is one of the four standard graph products [3] and is *the* product in the categorical sense, for instance in studies of graph mappings [2].

Jha [5] built r-perfect codes in the direct product of two cycles $C_{\ell_1} \times C_{\ell_2}$, where both ℓ_1 and ℓ_2 are multiples of $r^2 + (r+1)^2$. In another paper [4] he followed with a construction of r-perfect codes in the direct product of three cycles $C_{\ell_1} \times C_{\ell_2} \times C_{\ell_3}$, where each ℓ_i is a multiple of $r^3 + (r+1)^3$. Thus a natural question appears: what about products of cycles with more that three factors?

In this paper we extend the above Jha's results to any number of cycles by proving that for any $r \geq 1$, and any $n \geq 2$, each connected component of $\times_{i=1}^{n} C_{\ell_i}$ contains an r-perfect code provided that each ℓ_i is a multiple of $r^n + (r+1)^n$. In addition it is also proved that for $n = 2, 3, 4$ the direct product $\times_{i=1}^{n} C_{\ell_i}$ contains an r-perfect code if and only if each ℓ_i is a multiple of $r^n + (r+1)^n$ and it is conjectured that this is also true for $n \geq 5$.

We now formally define an r-perfect code. A set $C \subseteq V(G)$ is an *r-code* in G if $d(u,v) \geq 2r+1$ for any two distinct vertices $u, v \in C$. C is called an *r-perfect code* if for any $u \in V(G)$ there is exactly one $v \in C$ such that $d(u,v) \leq r$. Note that C is an 1-perfect code if and only if the closed neighborhoods of its elements form a partition of $V(G)$.

The *direct product* $G \times H$ of graphs G and H is the graph defined on the Cartesian product of the vertex sets of the factors. Two vertices (u_1, u_2) and (v_1, v_2) are adjacent whenever $u_1 v_1 \in E(G)$ and $u_2 v_2 \in E(H)$. For the graphs $G_1, \ldots, G_n$ we may write $G = G_1 \times \cdots \times G_n = \times_{i=1}^{n} G_i$ without parentheses and the vertices of G can be represented as vectors $\mathbf{v} = (v_1, \ldots, v_n)$, where $v_i \in V(G_i)$, $1 \leq i \leq n$.

For the cycle C_k $(k \geq 3)$ we will always assume $V(C_n) = \{0, 1, \ldots, k-1\}$. Whenever applicable, the computations will be done modulo k, that is, modulo the length of the appropriate cycle.

2 Results

Each connected component of the direct product of an arbitrary number of cycles contains an r-perfect code for any $r \geq 1$. For a given $r \geq 1$ we define $s = 2r+1$ and use this notation throughout the paper. For description of this perfect codes

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