Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
Electronic Notes in Discrete Mathematics 22 (2005) 261–265
Electronic Notes in DISCRETE MATHEMATICS
www.elsevier.com/locate/endm

Unicycle graphs and uniquely restricted maximum matchings

Vadim E. Levit[1]

Department of Computer Science
Holon Academic Institute of Technology
Holon, Israel

Eugen Mandrescu[2]

Department of Computer Science
Holon Academic Institute of Technology
Holon, Israel

Abstract

A matching M is called *uniquely restricted* in a graph G if it is the unique perfect matching of the subgraph induced by the vertices that M saturates. G is a *unicycle graph* if it owns only one cycle. Golumbic, Hirst and Lewenstein observed that for a tree or a graph with only odd cycles the size of a maximum uniquely restricted matching is equal to the matching number of the graph. In this paper we characterize unicycle graphs enjoying this equality. Moreover, we describe unicycle graphs with only uniquely restricted maximum matchings. Using these findings, we show that unicycle graphs having only uniquely restricted maximum matchings can be recognized in polynomial time.

Keywords: uniquely restricted matching, local maximum stable set, greedoid.

[1] Email: `levitv@hait.ac.il`
[2] Email: `eugen_m@hait.ac.il`

doi:10.1016/j.endm.2005.06.055

1 Introduction

Throughout this paper $G = (V, E)$ is a simple (i.e., a finite, undirected, loopless and without multiple edges) graph with vertex set $V = V(G)$ and edge set $E = E(G)$.

A graph is *unicycle* if it owns only one cycle.

A *stable* set in G is a set of pairwise non-adjacent vertices. A stable set of maximum size will be referred to as a *maximum stable set* of G, and the *stability number* of G, denoted by $\alpha(G)$, is the cardinality of a maximum stable set in G. By $\Omega(G)$ we mean the family of all maximum stable sets of the graph G.

A set $A \subseteq V(G)$ is a *local maximum stable set* of G if A is a maximum stable set in the subgraph spanned by $N[A]$, i.e., $A \in \Omega(G[N[A]])$, [6]. Let $\Psi(G)$ stand for the family of all local maximum stable sets of G. In [10], Nemhauser and Trotter Jr. showed that any local maximum stable set of a graph can be enlarged to one of its maximum stable sets.

A *matching* in a graph $G = (V, E)$ is a set of edges $M \subseteq E$ such that no two edges of M share a common vertex. A *perfect matching* is a matching saturating all the vertices of the graph.

A cycle C is *M-alternating* if for any two incident edges of C exactly one of them belongs to the matching M (see Kroghdal [5]). It is clear that an M-alternating cycle should be of even size.

A matching M in G is called *alternating cycle-free* if G has no M-alternating cycle. Alternating cycle-free matchings for bipartite graphs were first defined by Kroghdal in [5]. This kind of matchings was also investigated in connection with the so-called jump-number problem for partially ordered sets (see Chaty and Chein [1], Muller [9], Lozin and Gerber [8]).

A matching $M = \{a_ib_i : a_i, b_i \in V(G), 1 \leq i \leq k\}$ of a graph G is called *a uniquely restricted matching* if M is the unique perfect matching of the subgraph of G induced by $\{a_i, b_i : 1 \leq i \leq k\}$, (see Golumbic, Hirst, Lewenstein [2]). It appears also in the context of matrix theory, as a *constrained matching* (see Hershkowitz and Schneider [3]).

The matching number of G, denoted by $\mu(G)$, is the cardinality of a *maximum matching* (i.e., of a matching of maximum size). By $\mu_r(G)$ we mean the size of a maximum uniquely restricted matching of G. Clearly, $0 \leq \mu_r(G) \leq \mu(G)$ holds for any graph G.

Since any forest G, by definition, has no cycles, Theorem 1.1 ensures that all matchings of a forest are uniquely restricted, and therefore, $\mu_r(G) = \mu(G)$.