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Unions of perfect matchings in cubic graphs

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Abstract

We show that any cubic bridgeless graph with m edges contains two perfect matchings that cover at least $3m/5$ and three perfect matchings that cover at least $27m/35$ of its edges.

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1 Introduction

A well-known conjecture of Berge and Fulkerson states that every bridgeless cubic graph contains a family of six perfect matchings covering each edge exactly twice:

Conjecture 1.1 *Every cubic bridgeless graph G contains six perfect matching $M_1, \ldots, M_6$ such that each edge of G is contained in precisely two of the matchings.*

Conjecture 1.1 is attributed to Berge in [4], but it first appeared published in [3]. Cycle Double Conjecture is also closely related to this conjecture. Note also that Conjecture 1.1 trivially holds for cubic graphs G that are 3-edge-colorable.

The following weaker version of Conjecture 1.1 due to Berge is also open:

Conjecture 1.2 *Every cubic bridgeless graph G contains five perfect matchings $M_1, \ldots, M_5$ such that each edge of G is contained in at least one of the matchings.*

We remark that even if the number 5 in Conjecture 1.2 is replaced by any larger constant (independent of G), the statement is not known to be true. In this paper, we investigate the maximum possible size of the union of a given number of perfect matchings in a cubic bridgeless graph. More precisely, we study, for $k \in \{2, 3\}$, the numbers

$$m_k = \inf_G \max_{M_1,\ldots,M_k} \frac{|\bigcup_i M_i|}{|E(G)|},$$

where the infimum is taken over all bridgeless cubic graphs G, and $M_1, \ldots, M_k$ range over all perfect matchings of G. Note that Conjecture 1.2 asserts that $m_5 = 1$.

We determine the precise value of m_2 and provide a non-trivial lower bound on m_3. Let us begin by considering the upper bounds. The Petersen graph

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