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Optimal Ordering of Projections using Permutation Matrices and Angles between Projection Subspaces ⋆

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Abstract

This paper addresses the topic of the order of projection views when they are iteratively processed in a view-by-view manner. We investigate the 2D reconstruction problem with several processing order strategies. Numerical experiments giving a comparison of these strategies are presented.

Keywords: Order of Projection, Permutation Matrices, Golden Ratio.

1 Introduction

Early publications on iterative techniques for image reconstruction from projections pointed out the importance of the projection data order, that is, the sequence in which the available projections at different angles are accessed.

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However even now, there is no gold standard in this branch of tomography. To our knowledge, the candidates competing to be named as good ordering are: Random Access Scheme (RAS) [12], a Prime Number Decomposition (PND) principle [6], Multilevel Access Scheme (MAS) [4], Weighted Distance Scheme (WDS) [10] and Golden ratio-based (GR) ordering [8]. Together with those, the method of projection choice based on the seminal work of Hamaker and Solmon (HS) [5], is revisited in this work.

This paper tackles the problem of optimal ordering of parallel projections processed by iterative procedures which generate the 2-D reconstruction image on a projection-by-projection basis. We distinguish between continuous and discrete statements of the problem. By continuous formulation we mean that given image f, projections $p(\theta, s)$ are available (or can be calculated) for all values of variables $\theta \in [0, \pi), s \in [-1, 1]$. Then we seek to design a sequence $\theta_0, \theta_1, \theta_2, \dots$ of views (starting with $\theta_0 \equiv 0$) from interval $[0, \pi)$ which is good in terms of convergence of iterations, for all practically reasonable images f. The discrete formulation differs from the continuous problem only in that we are restricted to choose a certain ordering (permutation) within a set of N equally spaced projections. The best, or optimal ordering (for given image f) can be determined by time consuming global comparison of images reconstructed from differently ordered projections.

In this work we experiment numerically to find some figures-of-merit, or criteria, which indicate good orderings without exhaustive calculation of consecutive iterations. Ideally, we would like to construct a functional C which, applied to two different projection orderings P_1 and P_2, results in $||\mathcal{R}(P_1) - f|| \leq ||\mathcal{R}(P_2) - f||$ for all images f provided $C(P_1) \leq C(P2)$, where $\mathcal{R}(P)$ denotes reconstruction from the P - ordered data after few iterations i.e, a few cycles through the sequence. The discrete formulation of the problem is more close to practical situations; yet, the continuous case will reveal interesting properties of “self-organization” of projections in terms of the angles between corresponding Hilbert subspaces. Interesting properties of the Golden ratio-based order in terms of angles between projection subspaces are revealed. In the discrete formulation of the problem, we make an observation that the permutation matrix of the projection ordering and its eigenvalues behave with striking regularity for good or close to optimal projection orderings. We hypothesize that “good” permutation matrices can be categorized as $(0, 1)-$ matrices reconstructed from few “uniform” discrete projections.