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Prior Learning and Convex-Concave Regularization of Binary Tomography

Stefan Weber [a,1], Thomas Schüle [a,b,2], Christoph Schnörr [a,3]

[a] *University of Mannheim, Dept. M&CS, CVGPR-Group D-68131 Mannheim, Germany*

[b] *Siemens Medical Solutions, D-91301 Forchheim, Germany*

Abstract

In our previous work, we introduced a convex-concave regularization approach to the reconstruction of binary objects from few projections within a limited range of angles. A convex reconstruction functional, comprising the projections equations and a smoothness prior, was complemented with a concave penalty term enforcing binary solutions. In the present work we investigate alternatives to the smoothness prior in terms of probabilistically learnt priors encoding local object structure. We show that the difference-of-convex-functions DC-programming framework is flexible enough to cope with this more general model class. Numerical results show that reconstruction becomes feasible under conditions where our previous approach fails.

Keywords: Discrete Tomography, Markov Random Fields, Prior Learning, Convex-Concave Regularization

[1] Email:`wstefan@uni-mannheim.de`
[2] Email:`schuele@uni-mannheim.de`
[3] Email:`schnoerr@uni-mannheim.de`

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1 Introduction

1.1 Overview and Motivation

Discrete Tomography is concerned with the reconstruction of discrete-valued functions from projections. Historically, the field originated from several branches of mathematics like, for example, the combinatorial problem to determine binary matrices from its row and column sums (see the survey [12]). Meanwhile, however, progress is not only driven by challenging theoretical problems [7,10] but also by real-world applications where discrete tomography might play an essential role (cf. [11, chapters 15–21]).

The work presented in this paper is motivated by the reconstruction of volumes from *few* projection directions within a *limited* range of angles. From the viewpoint of established approaches to computational tomography [16], this is a severely ill-posed problem. The motivation for considering this difficult problem relates to the observation that in some specific scenarios [19] it is reasonable to assume that the function f to be reconstructed is *binary-valued*. This poses one of the essential questions of discrete tomography: how can knowledge of the discrete range of f be exploited in order to regularize and solve the reconstruction problem?

In our previous work [18], we introduced a convex-concave regularization approach to the binary reconstruction problem. Minimizing the squared residuals of the projection equations together with a smoothness prior favoring spatially homogeneous reconstructions was shown to considerably alleviate the ill-posedness of the reconstruction problem. Binary solutions were gradually computed in a " reconstruction-sensitive" way by *simultaneously* minimizing a concave penalty term. A primal-dual DC-programming algorithm particularly suited for this class of non-convex optimization problems showed promising performance.

Smoothness priors are convenient from the computational viewpoint because they result in *convex* functionals having relaxed the binary constraint. On the other hand, the signal class modelled thereby is limited to coarse-scale objects with large homogeneous areas or volumes. This motivates to investigate, within the same optimization framework, the use of priors encoding various object structures that have been probabilistically learnt from examples beforehand.