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Note

On inductively minimal geometries that satisfy the intersection property

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Abstract

We prove that, up to isomorphism, for a given positive integer *n*, there is only one inductively minimal pair $(\Gamma, Sym(n))$ of rank $n-1$ that satisfies the intersection property. Moreover, we show that the diagram of Γ is linear.

MSC: 51E24

Keywords: Incidence geometry; Inductively minimal; Intersection property

1. Introduction

Inductively minimal pairs (Γ, G) were introduced by Buekenhout in [2]. In [4], Buekenhout et al. classified these inductively minimal pairs. In [3], Buekenhout and Cara proved several properties of these pairs. In [8], Cara studied truncations of these inductively minimal pairs. Finally, in [9], Cara et al. counted these inductively minimal pairs up to isomorphism.

In [10], Jacobs and Leemans described algorithms to test the intersection property on coset geometries. Using these algorithms, they checked the intersection property on inductively minimal geometries up to $n = 6$. These geometries are available for instance in [5]. They

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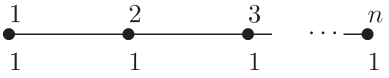
are the residually weakly primitive coset geometries of rank n with a connected diagram for the symmetric groups $Sym(n+1)$ (see [5] for the definitions).

It turned out that for each $n \leqslant 6$, up to isomorphism, only one inductively minimal geometry satisfies the intersection property. It is the one with a linear diagram. In this paper, we prove that if (Γ, G) is an inductively minimal pair and Γ satisfies the intersection property, then Γ is unique up to isomorphism and has the following diagram.



The paper is organised as follows. In Section 2, we recall some definitions and fix notation. In Section 3, we prove the result announced in this introduction.

2. Definitions and notation

We assume knowledge of the basic notions in incidence geometry as they are given for instance in [7] or [11].

Let $\Gamma(X, *, t, I)$ be an incidence geometry where X is the set of elements of Γ, $*$ is the incidence relation, t is the type function and I is the set of types of Γ. Given a type $i \in I$ and a flag F of Γ, we define the *i-shadow* $\sigma_i(F)$ as the set of elements of type i incident with F.

We define the intersection property (IP) as it appears in [1].

(IP) *For every type i, the intersection of the i-shadows of an element x and a flag F is either empty or equal to the i-shadow of a flag incident to x and F. The same holds on the residues.*

As mentioned in [6], this condition is equivalent to the following one.

(IP)′ *For every type i, the intersection of the i-shadows of an element x and a flag F is either empty or equal to the i-shadow of a flag incident to x and F.*

Let G be a group of automorphisms of Γ acting flag-transitively on Γ, that is, G acts transitively on the chambers of Γ.

As in [4] let (Γ, G) be called *minimal* if $\mid G \mid \leqslant (n+1)!$ where $n = \mid I \mid$. Let (Γ, G) be called *inductively minimal* if for any connected subset J of I and any flag F of Γ, with $t(F) = I \backslash J$, the pair (Γ_F, G_F), where G_F is the group induced on the residue Γ_F of the flag F in Γ by the stabilizer of F, is minimal.

3. Inductively minimal geometries and the intersection property

Buekenhout et al. show in [4] that a full control can be achieved on inductively minimal pairs although their number grows with n in a fairly wild way as it is shown in [9].