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Tableaux on $k + 1$ -cores, reduced words for affine permutations, and k -Schur expansions

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Abstract

The k -Young lattice Y^k is a partial order on partitions with no part larger than k . This weak subposet of the Young lattice originated (Duke Math. J. 116 (2003) 103–146) from the study of the k -Schur functions $s_\lambda^{(k)}$, symmetric functions that form a natural basis of the space spanned by homogeneous functions indexed by k -bounded partitions. The chains in the k -Young lattice are induced by a Pieri-type rule experimentally satisfied by the k -Schur functions. Here, using a natural bijection between k -bounded partitions and $k + 1$ -cores, we establish an algorithm for identifying chains in the k -Young lattice with certain tableaux on $k + 1$ cores. This algorithm reveals that the k -Young lattice is isomorphic to the weak order on the quotient of the affine symmetric group \tilde{S}_{k+1} by a maximal parabolic subgroup. From this, the conjectured k -Pieri rule implies that the k -Kostka matrix connecting the homogeneous basis $\{h_\lambda\}_{\lambda \in Y^k}$ to $\{s_\lambda^{(k)}\}_{\lambda \in Y^k}$ may now be obtained by counting appropriate classes of tableaux on $k + 1$ -cores. This suggests that the conjecturally positive k -Schur expansion coefficients for Macdonald polynomials (reducing to q, t -Kostka polynomials for large k) could be described by a q, t -statistic on these tableaux, or equivalently on reduced words for affine permutations.

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1. Introduction

1.1. The k -Young lattice

Recall that λ is a successor of a partition μ in the Young lattice when λ is obtained by adding an addable corner to μ where partitions are identified by their Ferrers diagrams, with rows weakly decreasing from bottom-to-top. This relation, which we denote “ $\mu \rightarrow \lambda$ ”, occurs naturally in the classical Pieri rule

$$h_1[X]s_\mu[X] = \sum_{\lambda: \mu \rightarrow \lambda} s_\lambda[X], \quad (1.1)$$

and the partial order of the Young lattice may be defined as the transitive closure of $\mu \rightarrow \lambda$. It was experimentally observed that the k -Schur functions [9,11] satisfy the rule

$$h_1[X]s_\mu^{(k)}[X] = \sum_{\lambda: \mu \rightarrow_k \lambda} s_\lambda^{(k)}[X], \quad (1.2)$$

where “ $\mu \rightarrow_k \lambda$ ” is a certain subrelation of “ $\mu \rightarrow \lambda$ ”. This given, the partial order of the k -Young lattice Y^k is defined as the transitive closure of $\mu \rightarrow_k \lambda$.

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