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2-Arc-transitive regular covers of complete graphs having the covering transformation group Z_p^3

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Abstract

A family of 2-arc-transitive regular covers of a complete graph is investigated. In this paper, we classify all such covering graphs satisfying the following two properties: (1) the covering transformation group is isomorphic to the elementary abelian p-group Z_p^3, and (2) the group of fiber-preserving automorphisms acts 2-arc-transitively. As a result, new infinite families of 2-arc-transitive graphs are constructed.

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1. Introduction

Throughout this paper, a graph is assumed to be finite, simple and undirected. For the group- and graph-theoretic terminology, we refer the reader to [17,20]. For a graph X, every edge of X gives rise to a pair of opposite arcs. By $V(X)$, $E(X)$, $A(X)$ and Aut X, we denote

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the vertex set, edge set, arc set and the full automorphism group of a graph X, respectively. The complete graph of order n is denoted by K_n. For any $v \in V(X)$, we use $N(v)$ to denote the neighborhood of v in X. A 2-*arc* of X is a sequence (v_0, v_1, v_2) of three vertices such that $(v_0, v_1), (v_1, v_2) \in A(X)$ but $v_0 \neq v_2$. The graph X is said to be 2-*arc-transitive* if Aut X acts transitively on the set of 2-arcs of X.

A graph X is called a *covering* (or *cover*) of a graph Y with the projection $p : X \to Y$ if there is a surjection $p : V(X) \to V(Y)$ such that $p|_{N(x)} : N(x) \to N(y)$ is a bijection for any vertex $y \in V(Y)$ and $x \in p^{-1}(y)$. The graph X is called the *covering graph* and Y is the *base graph*. A covering p is n-*fold* if $|p^{-1}(y)| = n$ for each $y \in V(Y)$. The *fiber* of an edge or a vertex is its preimage under p. An automorphism of X which maps a fiber to a fiber is said to be *fiber-preserving*. The group K of all automorphisms of X which fix each of the fibers setwise is called the *covering transformation group*. A cover X of Y is said to be *regular* (simply, L-*covering*) if there is a subgroup L of K acting freely and transitively (i.e., regularly) on each fiber. Moreover, if X is connected, then $L = K$.

This paper focuses on a classification of finite 2-arc-transitive graphs. The starting point for classifying such graphs is the following result of Praeger.

Proposition 1.1 (*Praeger [29, Theorem 4.1]*). *Let X be a connected graph and let G be a group of automorphisms of X which acts* 2*-arc-transitively on X. Assume that G has a normal subgroup N which has more than two orbits on $V(X)$. Let $\overline{X} = X/N$ denote the quotient graph of X by the N-action. Then, $\overline{X}$ is connected and N is semiregular on $V(X)$. Moreover, X is a regular cover of $\overline{X}$ with the covering transformation group N and G/N acts* 2*-arc-transitively on the quotient graph $\overline{X}$.*

By Proposition 1.1, the class of finite connected 2-arc-transitive graphs can be divided into two subclasses as follows:

(1) The 2-arc-transitive graphs X with the property that either (i) every nontrivial normal subgroup of Aut X acts transitively on $V(X)$, or (ii) every nontrivial normal subgroup of Aut X has at most two orbits on $V(X)$ and at least one of normal subgroups of Aut X has exactly two orbits on $V(X)$.

(2) The 2-arc-transitive regular covers of a graph given in (1).

A transitive permutation group G on a set Ω is said to be *primitive* if the property that for a subset $B \subset \Omega$, either $B^g = B$ or $B^g \cap B = \emptyset$ for each $g \in G$ is possible for only $B = \emptyset$, 1-element subset or the whole set Ω. It is *quasiprimitive* if every nontrivial normal subgroup of G is transitive on Ω. In particular, all primitive groups are quasiprimitive. However, there exist some quasiprimitive groups which are not primitive (see [29]). A structure theorem for finite quasiprimitive permutation groups was given in [29]. With these terminologies, in the first case (i) of the subclass (1), Aut X acts quasiprimitively on $V(X)$. During the last decades, many people have studied primitive or quasiprimitive finite 2-arc-transitive graphs (see [21,6–11] and others). In the second case (ii) of the subclass (1), X must be a bipartite graph and a reduction theorem for this case was given in [30]. In this paper, we examine only the second subclass (2).

Since the complete graph K_n is a standard example of a primitive 2-arc-transitive graph, it is interesting to investigate the *regular covers of K_n, when the group of fiber-preserving*