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Journal of Applied Logic 3 (2005) 421–460
JOURNAL OF APPLIED LOGIC
www.elsevier.com/locate/jal

Combinations of tense and deontic modality: On the R_t approach to temporal logic with historical necessity and conditional obligation

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Available online 3 June 2005

Editors: A. Lomuscio and D. Nute

Abstract

We consider three infinite hierarchies of what I call "two-dimensional temporal logics with explicit realization operators", viz. (i) one without historical or deontic modalities, (ii) one with historical but without deontic modalities, and (iii) one with historical and with dyadic deontic modalities for conditional obligation and permission. Sound and complete axiomatizations are obtained for all three hierarchies relative to a simplified version of the finite co-ordinate-system semantics given for so-called T × W logic of historical necessity in [L. Åqvist, The logic of historical necessity as founded on two-dimensional modal tense logic, J. Philos. Logic 28 (1999) 329–369].

Keywords: Temporal realization operators; Historical necessity; Conditional obligation; Finite two-dimensional co-ordinate system; Frame constants

1. Introduction

The purpose of this paper is to investigate some crucial properties of an infinite hierarchy of logics *combining* (i) a logic for the temporal realization operator R_t ["it is realized (true) at time t that"; see Rescher [17], Rescher and Urquhart [18]] *with* (ii) a modal logic for historical necessity or inevitability [Åqvist [3]], *and with* (iii) a dyadic deontic logic for conditional obligation Åqvist [2,4].

1570-8683/$ – see front matter
doi:10.1016/j.jal.2005.04.002

In order to provide some necessary background to our present enterprise, let us briefly consider the quite recent contribution Carmo and Jones [11, Section 7.1], where the authors make a number of useful observations concerning so-called *temporal approaches to the semantics of deontic notions* (like those of obligation and permission). The most important of these observations are, in my opinion, the following:

(I) The temporal approaches at issue are generally based on *tree-structures* representing branching time with the same past and open to the future.

(II) On top of these tree-structures, temporal deontic logics typically define one modal *necessity* operator, expressing some kind of *inevitability* or *historical* necessity, plus deontic *obligation* operators of either a monadic or dyadic kind (where the latter are to reflect notions of *conditional* obligation).

(III) A main difference appears in the way the temporal dimension is syntactically reflected in the formal language of the logics considered. One family of those logics *indexes* the modal and deontic operators with temporal *terms*, whereas another family introduces temporal *operators* that can be *iterated* and *combined with* the modal and deontic operators.

(IV) A characteristic feature of the "indexed" temporal deontic logics is then the presence in them of *time-indexed* modal and deontic operators: Carmo and Jones [11] point out that the time-index could be "separated" from the modal/deontic operators so as to yield a uniform semantical and logical setting for analyzing the modal/deontic component of both types of temporal deontic logics, mentioned in (III) above. This, they say, can be achieved by means of the temporal realization operator R_t ["it is realized (true) at time t that"] of Rescher and Urquhart [18]. Let us add here that this means that, instead of writing, like van Eck [12], Loewer and Belzer [15], and many others,

$N_t A$ for "it is necessary at time t that A",

$O_t A$ for "it is obligatory at time t that A", and

p_t for "p-at-time-t"

we are to write, following Bailhache [9,10] and myself in [5],

$R_t N A$,

$R_t O A$, and

$R_t p$

in order to express the corresponding notions, where the "separation" just spoken of is made perfectly clear and explicit.

In view of the above observations, the following problem naturally presents itself: What is the logic of the operators R_t, N, and O, considered (i) separately,[1] and (ii) in combination with one another? As for the logic of the modal operator N of historical necessity

[1] One should observe here that considering those logics "separately" does not preclude our basic two-dimensional temporal logic from containing *other* modal operators of great interest in their own right. See, e.g., Section 2 below *in fine*, category (vi).