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On the calculation of UNil

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Abstract

Cappell's codimension 1 splitting obstruction surgery group UNil_n is a direct summand of the Wall surgery obstruction group of an amalgamated free product. For any ring with involution R we use the quadratic Poincaré cobordism formulation of the L-groups to prove that

$$L_n(R[x]) = L_n(R) \oplus \mathrm{UNil}_n(R; R, R).$$

We combine this with Weiss' universal chain bundle theory to produce almost complete calculations of $\mathrm{UNil}_*(\mathbb{Z}; \mathbb{Z}, \mathbb{Z})$ and the Wall surgery obstruction groups $L_*(\mathbb{Z}[D_\infty])$ of the infinite dihedral group $D_\infty = \mathbb{Z}_2 * \mathbb{Z}_2$.

MSC: 57N15; 57R67

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0. Introduction

The nilpotent K- and L-groups of rings are a rich source of algebraic invariants for geometric topology, giving results of two types: if the groups are zero it is possible to solve the associated splitting and classification problems, while if they are nonzero

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the groups are infinitely generated and the solutions to the problems are definitely obstructed, see [2,4,6,8,9,10,11,16].

The unitary nilpotent L-groups UNil_* arise as follows. Suppose given a closed n-dimensional manifold X which is expressed as a union of codimension 0 submanifolds $X_1, X_{-1} \subseteq X$

$$X \;=\; X_1 \cup X_{-1}$$

with

$$X_0 \;=\; X_1 \cap X_{-1} \;=\; \partial X_{-1} \;=\; \partial X_1 \subseteq X$$

a codimension 1 submanifold. Assume X, X_{-1}, X_0, X_1 are connected, and that the maps $\pi_1(X_0) \to \pi_1(X_{\pm 1})$ are injective, so that by the van Kampen theorem the fundamental group of X is an amalgamated free product

$$\pi_1(X) \;=\; \pi_1(X_1) *_{\pi_1(X_0)} \pi_1(X_{-1})$$

with $\pi_1(X_i) \to \pi_1(X)$ $(i = -1, 0, 1)$ injective. Given another closed n-dimensional manifold M and a simple homotopy equivalence $f : M \to X$ there is a single obstruction

$$s(f) \in \mathrm{UNil}_{n+1}(R; \mathcal{B}_1, \mathcal{B}_{-1})$$

to deforming f by an h-cobordism of domains to a homotopy equivalence of the form

$$f_1 \cup f_{-1} \;:\; M_1 \cup M_{-1} \;\to\; X_1 \cup X_{-1}$$

with $f_{\pm 1} : (M_{\pm 1}, \partial M_{\pm 1}) \to (X_{\pm 1}, \partial X_{\pm 1})$ homotopy equivalences of manifolds with boundary such that

$$f_1| \;=\; f_{-1}| \;:\; \partial M_1 \;=\; \partial M_{-1} \to \partial X_1 \;=\; \partial X_{-1}$$

and

$$R \;=\; \mathbb{Z}[\pi_1(X_0)],\;\; \mathcal{B}_{\pm 1} \;=\; \mathbb{Z}[\pi_1(X_{\pm 1}) \backslash \pi_1(X_0)].$$

Cappell [5,6] proved geometrically that the free Wall [21] surgery obstruction groups $L_* = L^h_*$ of the fundamental group ring

$$\Lambda \;=\; \mathbb{Z}[\pi_1(X)] \;=\; \mathbb{Z}[\pi_1(X_1)] *_{\mathbb{Z}[\pi_1(X_0)]} \mathbb{Z}[\pi_1(X_{-1})]$$