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Simple p-kernels of p-divisible groups

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Dedicated to Mike Artin, for his 70th birthday

Abstract

A p-divisible group X determines its p-kernel $X[p] = G$. We show that G determines X uniquely if G is "minimal", and that there are infinitely many possibilities for X if G is not minimal. The indecomposable minimal G are precisly those which are simple.
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0. Introduction

In this paper we study p-divisible groups and their p-kernels over an algebraically closed field. We try to obtain an insight in which way these two kind of objects depend on each other. In this paper p is a prime number.

The p-kernel of a p-divisible group is called a BT_1 group scheme; here BT stand for Barsotti–Tate group scheme (a synonym for a p-divisible group), and the index 1 stands for "truncated at level one". For such group schemes we define the following notions: "indecomposable", see (1.2); "minimal", see (1.9); and "simple", see (1.2); this last terminology is short for the notion "BT_1-simple". With these notions defined we show:

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(0.1) Theorem A. *Let G be a* BT_1 *group scheme over an algebraically closed field* k;

$$G \text{ is indecomposable and minimal} \Longleftrightarrow G \text{ is simple}.$$

A proof will be given in Sections 3 and 5.

Starting from a p-divisible group X we obtain a BT_1 group scheme

$$[p] : \{X \mid \text{a } p\text{-divisible group}\}/\cong_k \to \{G \mid \text{a } \mathrm{BT}_1\}/\cong_k; \quad X \mapsto G := X[p].$$

This map is known to be surjective; see [2, 1.7], see [7, 9.10]; also see (2.5), where we define a section for this map. It is the main theorem of [9] that the fiber of this map over (G up to $\cong_k$) is precisely one p-divisible group X if G is minimal; however, things are different if G is not minimal, as we will show here:

(0.2) Theorem B. *Let G be a* BT_1 *group scheme over an algebraically closed field* k; *suppose G is not minimal*; *then*

$$\#(\{X \mid X[p] \cong G\}/\cong_k) = \infty.$$

A proof will be given in Sections 8–10.

We will see that there are subtle differences between numerical invariants attached to a p-divisible group X and to $X[p]$. This will make the proof of Theorem B not so easy. We illustrate this difficulty in an example in Section 6.

Throughout this paper we fix a prime number p. Base schemes will be over $\mathbb{F}_p$, and fields considered are of characteristic p. We use k and Ω for algebraically closed fields of characteristic $p > 0$.

We have gathered some information on notation in the first two sections; it seems best to start reading in Section 3, where the proofs start, and refer back whenever information on notation is needed.

1. Group schemes annihilated by p

We review results obtained in [4], and we fix notations (slightly different from notation used in that reference). We only study "circular words" as we do not need the "linear words" as in [1]. As we mainly study Dieudonné modules, the action of a word is given on the covariant Dieudonné modules of groups schemes studied (we had to make choices).

(1.1) BT_1 **group schemes.** A finite locally free group scheme $G \to S$ is called a BT_1 group scheme, see [2, p. 152], if $G[F] := \mathrm{Ker}\, F_G = \mathrm{Im} V_G =: V(G)$ and $G[V] = F(G)$. In particular, this implies that G is annihilated by p. The abbreviation "BT_1" stands for "1-truncated Barsotti–Tate group"; the terms "p-divisible group" and "Barsotti–Tate group" indicate the same concept. We refer to [1, Exp. VII_A.4], for the definition of $F_T : T \to T^{(p)}$ for an S-scheme $T \to S$ and of $V_G : G^{(p)} \to G$ for a commutative S-group scheme $G \to S$.

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