ELSEVIER

Bull. Sci. math. 129 (2005) 545–558

BULLETIN DES SCIENCES MATHÉMATIQUES

www.elsevier.com/locate/bulsci

Asymptotic behavior of alternative Jensen and Jensen type functional equations

John Michael Rassias [a,*], Matina John Rassias [b]

[a] *Pedagogical Department, E.E., National and Capodistrian University of Athens, Section of Mathematics and Informatics, 4, Agamemnonos str., Aghia Paraskevi, Athens 15342, Greece*

[b] *Statistics and Modelling Science, University of Strathclyde, Livingstone Tower, 26 Richmond str., Glasgow, UK, G1 1XH*

Received 17 February 2005

Available online 24 March 2005

Presented by David Eisenbud

Abstract

In 1941 D.H. Hyers solved the well-known Ulam stability problem for linear mappings. In 1951 D.G. Bourgin was the second author to treat the Ulam problem for additive mappings. In 1982–2005 we established the Hyers–Ulam stability for the Ulam problem of linear and nonlinear mappings. In 1998 S.-M. Jung and in 2002–2005 the authors of this paper investigated the Hyers–Ulam stability of additive and quadratic mappings on restricted domains. In this paper we improve our bounds and thus our results obtained, in 2003 for Jensen type mappings and establish new theorems about the Ulam stability of additive mappings of the second form on restricted domains. Besides we introduce alternative Jensen type functional equations and investigate pertinent stability results for these alternative equations. Finally, we apply our recent research results to the asymptotic behavior of functional equations of these alternative types. These stability results can be applied in stochastic analysis, financial and actuarial mathematics, as well as in psychology and sociology.

* Corresponding author.

E-mail addresses: jrassias@primedu.uoa.gr (J.M. Rassias), matina_rassias@yahoo.gr (M.J. Rassias), matina@stams.strath.ac.uk (M.J. Rassias).

URL: http://www.primedu.uoa.gr/~jrassias/.

0007-4497/$ – see front matter
doi:10.1016/j.bulsci.2005.02.001

Résumé

En 1941 D.H. Hyers a résolu le problème bien connu de stabilité d'Ulam pour les tracés linéaires. En 1951 D.G. Bourgin était le deuxième auteur pour traiter le problème d'Ulam pour les tracés additifs. En 1982–2005 nous avons établi la stabilité de Hyers–Ulam pour le problème d'Ulam des tracés linéaires et non-linéaires. En 1998 S.-M. Jung et en 2002–2005 les auteurs de cet article ont étudié la stabilité de Hyers–Ulam des tracés additifs et quadratiques sur des domaines restreints. Dans cet article nous améliorons nos limites et ainsi nos résultats obtenus, en 2003 pour le type tracés de Jensen et établissons de nouveaux théorèmes au sujet de la stabilité d'Ulam des tracés additifs de la deuxième forme sur des domaines restreints. Sans compter que nous présentons le type alternatif équations fonctionnelles de Jensen et étudions des résultats convenables de stabilité pour ces équations alternatives. En conclusion, nous appliquons nos résultats de la recherche récents au comportement asymptotique des équations fonctionnelles de ces types alternatifs. Ces résultats de stabilité peuvent être appliqués dans l'analyse stochastique, mathématiques financières et actuarielles,aussi bien qu'en la psychologie et la sociologie.

MSC: primary 39B; secondary 26D

Keywords: Ulam stability; Asymptoticity; Alternative Jensen; Alternative Jensen type; Restricted domain

1. Introduction

In 1940 and in 1964 S.M. Ulam [26] proposed the *general Ulam stability problem*:

"When is it true that by slightly changing the hypotheses of a theorem one can still assert that the thesis of the theorem remains true or approximately true?"

In 1941 D.H. Hyers [13] solved this problem for linear mappings. In 1951 D.G. Bourgin [3] was the second author to treat the Ulam problem for additive mappings. In 1978, according to P.M. Gruber [12], this kind of stability problems is of particular interest in probability theory and in the case of functional equations of different types. In 1980 and in 1987, I. Fenyö [7,8] established the stability of the Ulam problem for quadratic and other mappings. In 1987 Z. Gajda and R. Ger [10] showed that one can get analogous stability results for subadditive multifunctions. Other interesting stability results have been achieved also by the following authors J. Aczél [1], C. Borelli and G.L. Forti [2,9], P.W. Cholewa [4], St. Czerwik [5], and H. Drljevic [6]. In 1982–2005 J.M. Rassias [16–21,23,24] and in 2003 and 2005 the authors [22,25] solved the above Ulam problem for Jensen and Euler–Lagrange type mappings. In 1999 P. Gavruta [11] answered a question of ours [18] concerning the stability of the Cauchy equation. In 1998 S.-M. Jung [14] and in 2002–2003 the authors [21,22] investigated the Hyers–Ulam stability for additive and quadratic mappings on restricted domains. In this paper we improve our bounds and thus our results obtained, in 2003 for Jensen and Jensen type mappings and establish new theorems about the Ulam stability of additive mappings of the second form on restricted domains. Besides we introduce alternative Jensen and Jensen type functional equations and investigate pertinent stability results for these alternative functional equations. Finally, we