Available online at www.sciencedirect.com
SCIENCE DIRECT®
C. R. Acad. Sci. Paris, Ser. I 340 (2005) 759–764
COMPTES RENDUS MATHEMATIQUE
http://france.elsevier.com/direct/CRASS1/

Statistique/Systèmes dynamiques

Approche non paramétrique du filtrage de système non linéaire à temps discret et à paramètres inconnus

Vivien Rossi [a], Jean-Pierre Vila [b]

[a] *UMR I3M, équipe de probabilités et statistique, université Montpellier II, cc 51, place Eugène-Bataillon, 34095 Montpellier cedex 5, France*
[b] *UMR analyse des systèmes et biométrie, ENSAM-INRA, 2, place Pierre-Viala, 34060 Montpellier cedex 1, France*

Reçu le 28 octobre 2004 ; accepté après révision le 5 avril 2005

Présenté par Paul Deheuvels

Résumé

Les filtres particulaires sont actuellement les outils de filtrage de système non linéaire à temps discret les plus performants. Toutefois la présence de paramètres inconnus dans les équations du système rend leur mise en œuvre délicate et compromet souvent leur convergence. Cette Note montre comment une estimation en ligne convergente de ces paramètres peut être obtenue simultanément à celles des variables d'état à filtrer. Elle repose sur une méthode d'estimation non paramétrique de densités de probabilités conditionnelles par noyau de convolution, à partir de générations successives de particules du système. ***Pour citer cet article : V. Rossi, J.-P. Vila, C. R. Acad. Sci. Paris, Ser. I 340 (2005).***

Abstract

Filtering discrete time nonlinear system with unknown parameters: a non parametric approach. Particle filters are presently among the most powerful tools for filtering discrete time non linear systems. However the presence of unknown parameters in the system equations makes their use more complex and can even impair their convergence properties. This Note shows how an on-line consistent estimation of these parameters can be obtained simultaneously to that of the state variables to be filtered. This approach relies upon a kernel-based non parametric estimation of conditional probability densities from successive Monte Carlo generations of system particles. ***To cite this article: V. Rossi, J.-P. Vila, C. R. Acad. Sci. Paris, Ser. I 340 (2005).***
© 2005 Académie des sciences. Publié par Elsevier SAS. Tous droits réservés.

Adresses e-mail : rossiv@ensam.inra.fr (V. Rossi), vila@ensam.inra.fr (J.-P. Vila).

1631-073X/$ – see front matter
doi:10.1016/j.crma.2005.04.008

Abridged English version

This Note deals with the problem of filtering non linear discrete time system of the general form

$$\begin{cases} x_{t+1} = f_t(x_t, \theta_x, \varepsilon_t), \\ y_{t+1} = h_t(x_{t+1}, \theta_y, \eta_t) \end{cases} \tag{1}$$

in which f_t, h_t are known Borelian functions, $x_t \in \mathbb{R}^d$ is the vector of state variables, $y_t \in \mathbb{R}^q$ is the vector of observed variables, $\theta = (\theta_x^T, \theta_y^T)^T \in \Theta \subset \mathbb{R}^p$ a vector of fixed unknown parameters, ε_t, and η_t are vectors of independent white noises such that the density $p_t(y_t|x_t)$ exists and is bounded.

The first purpose of filtering is to estimate at each time t the probability distribution of the state vector x_t conditional on the observed variables up to time t, the so-called optimal filter, or if it exists its density function $p_t(x|y_1, \ldots, y_t)$. Among the most powerful methods for filtering nonlinear systems are the particle filters which proceed through successive Monte Carlo model simulations of state particles $\tilde{x}_t^i$, allowing a convergent empirical estimation of the state conditional probability distribution (Del Moral [2]). Typically in this family of particle methods, the evolution of the particles at each step is controlled through their likelihood with respect to the observed variables. These methods are thus dependent on the availability of the analytic form of this likelihood function and on the presence of non null observation noises. Moreover, when unknown parameters are present as in model (1) any filtering procedure has to take account of these parameters θ in the model equations and must be backed up with a simultaneous parameter estimation procedure. The estimation of θ can even be the main objective. Standard particle filter methods do not cope easily with this issue (Doucet et al. [6]).

In the following a convergent kernel-based non parametric particle procedure is proposed to perform estimation of both state and parameter vector conditional densities. It generalizes a state-dedicated non parametric conditional density estimation procedure recently developed (Rossi and Vila [13]). This new procedure relies first on the introduction into the system equations (1) of the natural model $\theta_{t+1} = \theta_t$ for the parameters whose possible values are endowed with an a priori probability density p_0^θ, the support of which is supposed to cover the true unknown parameter values θ^*. The system model (1) so completed is used to generate by simulation at each successive time step t, n particles $(\tilde{x}_t^i, \tilde{\theta}_t^i, \tilde{y}_t^i)$, $i = 1, \ldots, n$, from which and from the current observed y_t value, kernel-based conditional density estimates of the state vector x_t and the parameter vector θ_t are built up. The recursive use of these successive density estimates in the particle generation from step to step, ensures their conditioning on the successive observations up to the one at hand.

For a given time step t, almost sure L_1-convergence results of these conditional density estimates to their true counterparts, as the number n of generated particles grows to infinity, are established with their rate of convergence. These L_1-convergence results have no equivalent in the standard particle filter methods which can only proceed with discrete approximations of conditional probability measures. Most remarkably, these convergence results are obtained under simple assumptions as $\lim_{n\to\infty} nh_n^{d+p+q}/\log n = \infty$ and $\lim_{n\to\infty} h_n = 0$ with h_n the kernel bandwidth parameter, which are independent of the time step t.

Moreover, almost sure convergence of conditional expectation estimates $\hat{x}_t^n$ and $\hat{\theta}_t^n$ of the state vector x_t and the parameter vector θ as n grows to infinity, are also provided.

Finally, a direct application of a theorem of Schervish [14] ensures that the conditional θ_t probability density function concentrates around the true unknown parameter value θ^* as t grows to infinity and then the same is true for its filter estimate when in addition n tends also to infinity. In a similar way the direct application of a theorem of Schwartz [15] ensures that the conditional θ_t expectation $\mathbb{E}[\theta_t|y_{1:t}]$, converges to θ^* almost surely as t tends to infinity and the same is true for its filter estimate $\hat{\theta}_t^n$ when in addition n grows to infinity.

1. Introduction

Les équations d'état et d'observation d'un système dynamique contiennent souvent des paramètres fixes mais inconnus. Un tel contexte est formalisé par le système (1). L'idée d'utiliser des méthodes de filtrage particulaire