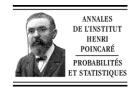


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# Green kernel estimates and the full Martin boundary for random walks on lamplighter groups and Diestel–Leader graphs <sup>☆</sup>

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#### Abstract

We determine the precise asymptotic behaviour (in space) of the Green kernel of simple random walk with drift on the Diestel–Leader graph DL(q, r), where  $q, r \ge 2$ . The latter is the horocyclic product of two homogeneous trees with respective degrees q + 1 and r + 1. When q = r, it is the Cayley graph of the wreath product (lamplighter group)  $\mathbb{Z}_q \wr \mathbb{Z}$  with respect to a natural set of generators. We describe the full Martin compactification of these random walks on DL-graphs and, in particular, lamplighter groups. This completes previous results of Woess, who has determined all minimal positive harmonic functions. © 2005 Elsevier SAS. All rights reserved.

#### Résumé

On détermine le comportement asymptotique précis (dans l'espace) du noyau de Green de la marche aléatoire simple avec dérive sur le graphe de Diestel-Leader DL(q, r), où  $q, r \ge 2$ . Ce graphe est le produit horocyclique de deux arbres homogènes de degrés q + 1 et r + 1, respectivement. Quand q = r, il s'agit du graphe de Cayley du produit en couronne (« lamplighter group »)  $\mathbb{Z}_q \wr \mathbb{Z}$  par rapport à un ensemble naturel de générateurs. On décrit la compactification de Martin complète de ces marches aléatoires sur les graphes DL, et en particulier, les groupes du « lamplighter ». Ceci complète les résultats précédents de Woess, qui a déterminé les fonctions harmoniques minimales. © 2005 Elsevier SAS. All rights reserved.

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### 1. Introduction

Consider the additive group  $\mathbb{Z}$  of all integers as a two-way-infinite road where at each point there is a lamp that may be switched on in one of q different intensities (states)  $\in \mathbb{Z}_q = \{0, \ldots, q-1\}$ , the group of integers modulo q. At the beginning, all lamps are in state 0 (switched off), and a lamplighter starts at some point of  $\mathbb{Z}$ . He chooses at random among the following actions (or a suitable combination thereof): he can move to a neighbour point in  $\mathbb{Z}$ , or he can change the intensity of the lamp at the actual site to a different state. As the process evolves, we have to keep track of the position  $k \in \mathbb{Z}$  of the lamplighter and of the finitely supported configuration  $\eta : \mathbb{Z} \to \mathbb{Z}_q$  that describes the states of all lamps. The set  $\mathbb{Z}_q \wr \mathbb{Z}$  of all pairs  $(\eta, k)$  of this type carries the structure of a semi-direct product of  $\mathbb{Z}$  with the additive group C of all configurations, on which  $\mathbb{Z}$  acts by translations. This is often called the *lamplighter group*; the underlying algebraic construction is the *wreath product* of two groups.

Random walks on lamplighter groups have been a well-studied subject in recent years, see Kaimanovich and Vershik [18] and Kaimanovich [17] (Poisson boundary  $\equiv$  bounded harmonic functions), Lyons, Pemantle and Peres [20], Erschler [12], Revelle [24], Bertacchi [3] (rate of escape), Grigorchuk and Żuk [13], Dicks and Schick [7], Bartholdi and Woess [2] (spectral theory), Saloff-Coste and Pittet [22,23], Revelle [25] (asymptotic behaviour of transition probabilities), and Woess [28] (positive harmonic functions).

Here, we shall deal with Green kernel asymptotics and positive harmonic functions. Let us briefly outline in general how this is linked with *Martin boundary theory* of Markov chains. Consider an arbitrary infinite (connected, locally finite) graph X (e.g., a Cayley graph of a finitely generated group) and the stochastic transition matrix  $P = (p(x, y))_{x,y \in X}$  of a random walk  $Z_n$  on X. That is,  $Z_n$  is an X-valued random variable, the position of the random walker at time n, subject to the Markovian transition rule

$$\Pr[Z_{n+1} = y \mid Z_n = x] = p(x, y).$$

The *n*-step transition probability

$$p^{(n)}(x, y) = \Pr[Z_n = y \mid Z_0 = x], \quad x, y \in X,$$

is the (x, y)-entry of the matrix power  $P^n$ , with  $P^0 = I$ , the identity matrix. The *Green kernel* is

$$G(x, y) = \sum_{n=0}^{\infty} p^{(n)}(x, y), \quad x, y \in X.$$

This is the expected number of visits in the point y, when the random walk starts at x. We always consider random walks that are *irreducible* and *transient*, which amounts to

 $0 < G(x, y) < \infty$  for all  $x, y \in X$ .

*Renewal theory* in a wide sense consists in describing the asymptotic behaviour in space of G(x, y), when x is fixed and y tends to infinity (or dually, y is fixed and x tends to infinity). If we fix a reference point  $o \in X$ , then the *Martin kernel* is

$$K(x, y) = G(x, y)/G(o, y), \quad x, y \in X.$$

If we have precise asymptotic estimates in space of the Green kernel, then we can also determine the *Martin* compactification. This is the smallest metrizable compactification of X containing X as a discrete, dense subset, and to which all functions  $K(x, \cdot), x \in X$ , extend continuously. The *Martin boundary*  $\mathcal{M} = \mathcal{M}(P)$  is the ideal boundary added to X in this compactification. Thus,  $\mathcal{M}$  consists of the "directions of convergence" of K(x, y), when  $y \to \infty$ . Its significance is that it leads to a complete understanding of the cone  $\mathcal{H}^+ = \mathcal{H}^+(P)$  of positive harmonic functions. A function  $h: X \to \mathbb{R}$  is called harmonic, or *P*-harmonic, if

$$h = Ph$$
, where  $Ph(x) = \sum_{y} p(x, y)h(y)$ .

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