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Finite utility on financial markets with asymmetric information and structure properties of the price dynamics ☆

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Abstract

We consider financial markets with two kinds of small traders: regular traders who perceive the (continuous) asset price process S through its natural filtration, and insiders who possess some information advantage which makes the filtrations through which they experience the evolution of the market richer. We discuss the link between (NFLVR), the semimartingale property of S viewed from the agent's perspective, and bounded expected utility. We show that whenever an agent's expected utility is finite, S is a semimartingale with a Doob–Meyer decomposition featuring a martingale part and an information drift. The expected utility gain of an insider with respect to a regular trader is calculated in a completely general setting. In particular, for the logarithmic utility function, utility gain is a function of the relative information drift alone, regardless of whether the market admits arbitrage.

Résumé

On considère un marché financier avec deux sortes de petits investisseurs : des investisseurs réguliers qui perçoivent l'évolution du prix S dans sa filtration naturelle, et des agents initiés ayant des informations supplémentaires et qui ainsi suivent l'évolution du prix par une filtration plus riche. On discute le rapport entre (NFLVR), la propriété de semimartingale de S vue dans la perspective de l'agent, et la bornitude de l'espérance de l'utilité. Si celle-ci est bornée, S est une semimartingale avec une décomposition de Doob–Meyer comprenant une martingale et un drift d'information. On calcule l'utilité supplémentaire espérée pour l'agent initié sous des conditions générales. On montre, que si la fonction d'utilité est logarithmique, l'utilité supplémentaire ne dépend que du drift d'information. C'est le cas même si le marché admet un arbitrage.

MSC: primary 60G48, 60H05, 91B28; secondary 93E20, 94A17, 60H30

Keywords: Insider trading; Enlargement of filtration; Free lunch with vanishing risk; (NFLVR); Arbitrage; Finite expected utility; Semimartingale; Stochastic integrator; Information drift

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0. Introduction

Asymmetry of information on financial markets has been a subject of increasing interest in recent years. Several mathematical models have been designed to deal with financial markets on which traders with different information levels are active. See Wu [32] for an overview. The model to capture basic facts of insider's action on markets which motivated this paper is very simple. Two kinds of traders are considered: regular agents who do not know any more than the natural evolution of the assets of the market, and insiders whose knowledge at any given time in the trading interval is larger than the σ-field generated by the asset price process up to that time. The insider may, for example, possess some additional information on the price of an asset at maturity, or at some later time. He might anticipate the time when an asset price reaches a favorable level, or be able to stop at the time at which some final level crossing of the price process occurs. Situations of this type have been modelled for example by Karatzas, Pikovsky [22], Amendinger [1], Amendinger, Becherer and Schweizer [2], Grorud, Pontier [16], and [3,19,17,18]. In most of these papers, questions of utility gain of the insider relative to the regular trader were discussed. It turned out that for many types of additional information the expected increment of utility gained by the insider may become infinite quite easily, and might provide opportunities for free lunch or even arbitrage in an equally easy way. Baudoin [5,6] and Baudoin, Nguyen-Ngoc [7] develop a model in which additional information on some random variable unknown to the regular trader is only weakly available, i.e. in the form of some knowledge of its law instead of the precise anticipation of its value. In this framework the insider's utility is more likely to be finite and can be computed for example by means of the fundamental results by Kramkov, Schachermayer [23]. In [10], the precise observation of some random element by the insider which is inaccessible to the natural trader is blurred dynamically by some exterior independent noise to produce a weaker information advantage in the same spirit, and keep the additional utility from getting out of control.

A natural mathematical toolbox to use in the context of the models described contains the techniques of *grossissement de filtrations* developed in some deep work mostly by French authors [9,20,21,25,33–36,31]. This is just one of numerous examples in which the direct impact of Meyer's Strasbourg school on contemporary financial mathematics becomes evident. Another example is initiated in a recent paper by Biagini and Oksendal [8]. In this paper a question is raised which appears to be of purely mathematical interest at first glance: knowing that the expected utility of an insider is finite, what can be said about the regularity of the asset price process from the insider's point of view? The authors show that given finite utility and the existence of an optimal investment strategy for the insider, the asset price process must be a semimartingale in the insider's enlarged filtration. This way, they address one of the basic questions of the theory of grossissements de filtrations, and at the same time raise a problem which goes to the heart of stochastic analysis: the relationship between semimartingales and the stochastic integrator property. To describe the utility of the insider in his enlarged filtration, they use extended notions of stochastic integrals investigated in anticipative stochastic calculus, such as Skorokhod's integral (see Nualart [26]) and the forward Itô integral introduced by Russo and Vallois [29].

The deep and central theorem of Bichteler–Dellacherie–Mokobodski characterizes semimartingales as *good stochastic integrators*. A process S is a semimartingale if and only if the stochastic integrals of uniformly bounded simple processes, i.e. predictable step processes, with respect to S form a bounded set in the topological vector space of random variables with the (L^0-)topology of convergence in probability. This key theorem allows to deal with the problem posed by Biagini and Oksendal [8] from a different perspective. Suppose an agent invests on a financial market with asset price process S and measures the utility of his final wealth through a utility function U which is unbounded. Then the hypothesis that the expected utilities the agent is able to attain be a bounded function of the simple investment strategies he is allowed to use due to his information horizon should be closely related to the L^0-boundedness of the set of stochastic integrals of simple admissible strategies. Hence the theorem should indicate a direct link between finite utility of agents on financial markets and the semimartingale property of the asset price processes with respect to the evolution of their information. This basic observation is the starting point for the analysis presented in this paper.