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Estimating critical stream power for bedload transport calculations in gravel-bed rivers

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Abstract

The physical basis of Bagnold's equation for critical stream power is examined. Although the general approach is well founded, the original equation can be criticised for (1) failing to distinguish between the grain size in transport and the grain size that represents bed roughness; (2) requiring a knowledge of critical flow depth in addition to gross channel properties; (3) failing to allow for hiding and protrusion effects on the mobility of mixed-size stream beds. More general equations are presented which overcome these limitations. They require a knowledge of channel slope, but not depth. The analysis suggests that the presence of form resistance in addition to grain roughness does not affect the calculation of critical power. Critical power is expected to depend more sensitively on grain size between reaches than within reaches.

Keywords: Sediment transport; Entrainment; Threshold

1. Introduction

Bedload transport in gravel-bed rivers is commonly assumed to occur at a very low rate up to a certain critical level of streamflow, then to increase at a faster-than-linear rate with flow above this threshold. The flow variable used to predict bedload transport is usually shear stress, as for example in the well known equation of Meyer-Peter and Müller (1948). In uniform flow the mean shear stress, averaged across the channel, depends on the depth-slope product:

$$\tau = \rho g d S \tag{1}$$

where ρ denotes the density of water, g the acceleration due to gravity, d the mean flow depth, and S the bed and water-surface gradient. Bagnold (1977, 1980) proposed that bedload transport could alternatively be predicted from the mean value of stream power per unit bed area, defined as

$$\omega = \rho g Q S / w = \tau U \tag{2}$$

where Q denotes the water discharge, w the width of the river, and U the mean velocity. It should be noted that Bagnold omitted g in his equations; this has led to much confusion in the subsequent literature. Eq. (2) quantifies the rate of loss of potential energy as water flows downhill, and thus the power potentially available for performing geomorphic work. Bagnold

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proposed that bedload transport rate increases non-linearly with stream power above a threshold or critical value which I shall denote by ω_c.

Bagnold's approach is conceptually attractive insofar as it treats rivers as transporting (and therefore work-performing) machines with explicit attention to their efficiency. It is also pragmatically convenient in that stream power can be calculated from gross channel properties (width and slope), together with the discharge provided by the catchment, without needing to know within-channel flow properties such as depth or velocity. Discharge is essentially constant from one junction to the next along a river and may be known from hydrometric records or predictable from a hydrological model, whereas depth (needed for the calculation of shear stress) is locally variable and not routinely measured. Partly for these reasons, and also because it has performed well in some comparative tests (Gomez and Church, 1989; Martin and Church, 2000), Bagnold's transport equation has been used quite widely in the geomorphology literature.

There has been some re-evaluation of the efficiency factor in Bagnold's transport equation (e.g. Bagnold, 1980; Martin and Church, 2000), but all users of any version of the equation appear to have accepted Bagnold's (1980) proposal for specifying the critical power in terms of river properties:

$$\omega_c = c_1 D^{1.5} \log(c_2 d/D) \quad (3)$$

where c_1 and c_2 are numerical constants, D denotes the diameter of mobilised particles, and the logarithm is to base 10. Bagnold (1980) gave $c_1=290$ and $c_2=12$ without fully explaining his derivation. These values are for D in the same units as d, for example metres; and, as noted below, the predicted values of ω_c are not in the standard units of W m^{-2}.

The lack of attention to the specification of the threshold is surprising considering that all gravel transport equations are extremely sensitive to the threshold value of whichever flow variable is used, and that there is an extensive literature on threshold shear stresses for poorly sorted river beds (see for example Andrews, 1983; Komar, 1987; Ashworth and Ferguson, 1989). Petit et al. (2005) take an important first step towards assessing whether Eq. (3) is universally valid. They present a useful and interesting synthesis of empirically derived critical specific stream power values for streams in Belgium, and compare them with a version of Eq. (3). Their results are based on the movement of tracer pebbles in 14 streams and rivers with slopes ranging from <0.002 to 0.05. The inferred values of critical power show clear correlations with D within each data set, as expected from Eq. (3), but with displacements between data sets. Petit et al. (2005) speculate that these displacements reflect the varying importance of bedform resistance to flow in small, medium, and large rivers. Critical stream power was found to be higher in channel styles where a higher proportion of the total shear stress (as given by Eq. (1)) is dissipated in overcoming bedform resistance.

This note was inspired by reading the work of Petit et al. (2005) but is largely complementary to it. I consider the question of critical stream power from a theoretical point of view. I show that Bagnold's expression for critical power can be criticised on several grounds, and present alternative equations based on a more detailed analysis which overcomes these problems. I then discuss the implications for differences in critical power within and between reaches. I argue on physical grounds that critical power is not affected by form resistance, but is higher in reaches with coarser beds and also varies somewhat with channel slope; this provides an alternative explanation for the pattern of differences found by Petit et al. (2005). A detailed comparison with their data is not possible at this time for lack of sufficient published detail about the characteristics of their individual reaches. The theoretical rate of increase of critical power with grain size within reaches is less than suggested by the empirical curves presented by Petit et al. and previous workers, but the discrepancy is mainly due to their aggregation of results from different reaches.

2. Derivation of general equations for critical stream power

Bagnold (1980) derived Eq. (3) from the identity $\omega_c = \tau_c U_c$ by relating τ_c to D through Shields' criterion for incipient motion, and relating U_c to d/D using a flow resistance equation of logarithmic type. He gives few further details of the derivation but it is fairly easy to reconstruct. Before doing so, though, a first criticism of Bagnold's equation is