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# Microprocessor-based simulation of sampled data systems with/without a hold device using a set of sample-and-hold functions and Dirac delta functions

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**Abstract**

In the present work, Dirac delta function (DF) set and sample-and-hold functions (SHF) set are used for microprocessor-based simulation of discrete time as well as sample-and-hold systems. Such simulations are useful for identification of control systems with known input and output sequence. The presented method utilizes operational matrices of different orders in the DF and SHF domain to develop different operational transfer functions. A few open-loop as well as closed-loop systems have been studied and the simulation results obtained are compared with exact solutions derived with the help of $z$-transform analysis. Experiments have also been carried out to establish the validity of the proposal.


*Keywords:* Microprocessor; Sampled-data systems; Orthogonal functions

## 1. Introduction

Piecewise constant basis functions (PCBF) have a very rich history [1] starting from the year 1910. In 1973, Corrington [2] employed Walsh functions for solving

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differential and integral equations, thus breaking the ice and paving the way to system analysis. Definition and some applications of functions of this class, e.g., Haar function, Walsh function, block pulse function (BPF), etc., to systems and control are discussed elaborately in Ref. [3].

BPF were first introduced in the literature in 1969 by Harmuth [4]. But its application to system analysis started only in early 1980's. With progress in research in the area of such kind of orthogonal functions, it was found that the BPF was the simplest as well as the most efficient [5] amongst its sister functions. Naturally, it became a popular effective analysis tool in the area of control theory within a short time. Ref. [6] is dedicated to block pulse function and its application to control system analysis and identification.

Later, it was shown by Deb et al. [7] that analysis with a newly defined pulse width modulated block pulse function (PWM-BPF) set used less number of component functions compared with the unmodulated one.

A later work [8] involving the BPF set derived different operational transfer functions with the help of repeated integration using one-shot operational matrices [3].

Other than BPF, a few more new function sets of related nature were introduced by Deb et al. [9,10] and successfully applied to discrete time systems and systems with sample-and-hold.

In the present work, Dirac delta function set and sample-and-hold function set are used for microprocessor-based simulation of discrete time as well as sample-and-hold systems. A few open-loop and closed-loop sampled data systems—with/without hold device—have been studied and the results obtained are compared with exact solutions.

## 2. Review of delta function and sample-and-hold function operational technique

### 2.1. Delta function domain operational technique [9]

If a square integrable time function $f(t)$ is fed to a sampling device, the device modulates $f(t)$ to $f^*(t)$ which is obtained as the output. Hence

$$\begin{aligned} f^*(t) &= \sum_{i=0}^{m-1} f_i \delta_i(t) \\ &= [f_0 \; f_1 \; f_2 \ldots f_i \ldots f_{(m-1)}] \boldsymbol{\Delta}_{(\mathbf{m})}(\mathbf{t}) \\ &\triangleq \mathbf{F}_{(\mathbf{m})} \boldsymbol{\Delta}_{(\mathbf{m})}(\mathbf{t}), \end{aligned} \tag{1}$$

where we have chosen an $m$-set Dirac delta functions given by

$\boldsymbol{\Delta}_{(\mathbf{m})}(\mathbf{t}) \triangleq [\delta_0(t) \;\; \delta_1(t) \ldots \delta_i(t) \ldots \delta_{(m-1)}(t)]^{\mathrm{T}}, [\ldots]^{\mathrm{T}}$ denoting transpose and the component delta functions are delayed delta functions given by

$$\delta_i(t) \triangleq \delta(t - ih),$$