Available online at www.sciencedirect.com
SCIENCE DIRECT
Journal of the Franklin Institute 342 (2005) 674–687
Journal of The Franklin Institute
www.elsevier.com/locate/jfranklin

Frequency-domain approach to robust PI control

Wieslaw Krajewski[a], Antonio Lepschy[b], Stefano Miani[c,*], Umberto Viaro[c]

[a]Systems Research Institute, Polish Academy of Sciences, ul. Newelska 6, 01-447 Warsaw, Poland
[b]Department of Information Engineering, University of Padova, Via Gradenigo 6/b, 35131 Padova, Italy
[c]Department of Electrical, Management and Mechanical Engineering, University of Udine, Via delle Scienze 208, 33100 Udine, Italy

Abstract

The paper considers the problem of designing PI controllers for industrial processes approximated by a first-order time-delayed model. The suggested frequency-domain approach is based on a normalized open-loop transfer function and makes use of the loci of constant stability margins and other performance indices in the parameter space. In this way, it is possible to evaluate the effects of uncertainties in the process parameters and, thus, control system robustness. Some examples show how the procedure operates.

Keywords: Process control; Robustness; PI controllers; Time-delayed processes; Stability margins

1. Introduction and preliminaries

The large majority of industrial control loops resort to PID controllers [1,2] and the plants to be controlled are adequately approximated by a first-order lag with time delay (FOLTD) model (cf., e.g., [3,4] and bibliographies therein) whose transfer

*Corresponding author. Tel.: +39 0432 558262; fax: +39 0432 558251.
E-mail address: miani.stefano@uniud.it (S. Miani).

0016-0032/$30.00
doi:10.1016/j.jfranklin.2005.04.003

function is

$$P(s) = K\frac{e^{-Ls}}{1 + Ts}. \tag{1}$$

In fact, industrial processes are often characterized by a monotonically increasing S-shaped step response like the one generated by an all-pole transfer function or by a transfer function with a high pole-zero excess in which the poles and the zeros, if any, are (negative) real. Since the step response of Eq. (1) is equal to zero for $t < L$ and tends monotonically to K as $t \to \infty$, it can be reasonably adopted to approximate the behaviour of the afore-mentioned systems, especially for the purpose of designing a controller of simple structure like a PI controller whose transfer function is

$$C(s) = K_P + \frac{K_I}{s} = K_P\left(1 + \frac{1}{T_I s}\right). \tag{2}$$

Of course, K, L and T in Eq. (1) must be preliminarily estimated using suitable techniques (among the simplest ones, the so-called area method proves to be quite satisfactory [3,5]). Indeed, the two fundamental pillars of PID controller synthesis are: model parameter estimation and controller parameter tuning. Many textbooks and survey papers (cf., e.g., [4,6]) are devoted to either aspect. In the following, we focus attention on the latter.

Various "recipes" relating parameters K_P and K_I in Eq. (2) to those in Eq. (1) have been proposed to tune PID controllers since the pioneering paper by Ziegler and Nichols [7] (more recently, *automatic* tuning procedures have been considered with increasing interest [8]). Here, we follow the alternative path of determining the controller parameters so as to meet given design specifications. In particular, this paper is concerned with the synthesis of a PI controller for a plant like Eq. (1) according to frequency-domain specifications that account for both robustness and performance.

Concerning robustness, in the literature reference has been made to standard stability margins [9], complex stability margins [10], structured singular values [11] or quantitative feedback theory (QFT) (cf. [12]). In the next sections we consider various types of stability margins.

Concerning performance, we refer, as usual, to step response overshoot and rise time, which are strictly related to phase margin (cf., e.g., [13]) and gain cross-over frequency.

To simplify the design procedure, it is particularly convenient to use the open-loop transfer function:

$$G(s) = \frac{(a + bs)e^{-\tau s}}{s(1 + s)}, \tag{3}$$

where frequencies have been normalized to $1/T$, so that the relations between the parameters in Eq. (3) and those in Eqs. (1) and (2) are:

$$a := K_I K T, \quad b := K_P K, \quad \tau = \frac{L}{T}. \tag{4}$$