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New design method on memoryless H_∞ control for singular systems with delayed state and control using LMI

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Abstract

This note presents a new memoryless H_∞ controller design method for singular systems with delayed state and control by linear matrix inequality (LMI) technique. Since lemma and theorem in the paper by Hung and Lee (J. Franklin Inst. 336 (1999) 911–923) are incorrect, the errors are pointed out. And then, a new state feedback H_∞ controller design algorithm for singular systems with delayed state and control by LMI approach. All solutions including controller gain can be obtained simultaneously because the presented condition is an LMI regarding all variables.

Keywords: Singular system; H_∞ control; Time delay; Linear matrix inequality

1. Introduction

Recently, the singular H_∞ control problem has been widely studied due to the fact that singular systems better describe physical systems than regular ones. Also, H_∞ control for time delay systems has been an issue of recurring interest over the past decades since time delays are often the main causes for instability and poor performance of systems and encountered in various engineering systems such as

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chemical processes, long transmission lines in pneumatic systems, and so on [1]. Recently, Hung and Lee [2] proposed a method of H_∞ controller design of singular systems with delayed state and control based on the Riccati equation approach. However, there are errors in the paper [2] as follows.

Comment 1. The condition in Lemma 2 of [2] cannot be satisfied for any positive symmetric matrices P, R and positive scalar ε. For a vector $x(t)\neq 0$ such that $Ex(t)=0$, the left-hand side of (14) in [2] implies

$$x(t)^{\mathrm{T}}\Big[E^{\mathrm{T}}PA + A^{\mathrm{T}}PE - \frac{1}{\varepsilon}E^{\mathrm{T}}PBR^{-1}B^{\mathrm{T}}PE + \frac{1}{\varepsilon}E^{T}PB_{\mathrm{d}}R^{-1}B_{\mathrm{d}}^{\mathrm{T}}PE + E^{\mathrm{T}}PA_{\mathrm{d}}A_{\mathrm{d}}^{\mathrm{T}}PE + I_n\Big]x(t) = x(t)^{\mathrm{T}}x(t), \tag{1}$$

which is nonnegative. Therefore, Lemma 2 of [2] is incorrect.

Proof. To prove the error, we consider a counterexample. If we take the same matrices and values of variables in example [2] and define state vector as follows:

$$x(t) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{2}$$

Then, the left-hand sides of Eq. (1) implies

$$x(t)^{\mathrm{T}}\Big[E^{\mathrm{T}}PA + A^{\mathrm{T}}PE - \frac{1}{\varepsilon}E^{\mathrm{T}}PBR^{-1}B^{\mathrm{T}}PE + \frac{1}{\varepsilon}E^{\mathrm{T}}PB_{\mathrm{d}}R^{-1}B_{\mathrm{d}}^{\mathrm{T}}PE + E^{\mathrm{T}}PA_{\mathrm{d}}A_{\mathrm{d}}^{\mathrm{T}}PE + I_n\Big]x(t) = 1. \tag{3}$$

This is not negative. Hence, Eq. (19) in [2] cannot be rewritten as Eq. (22) in [2] because the specific value of $x(t)$ cannot make the condition negative. Therefore, the sufficient condition (14) in Lemma 2 [2] is not true. □

Comment 2. Similarly, it is not possible to hold for condition (23) in Theorem 1 of [2]. Repeatedly, for a vector $x(t)\neq 0$ such that $Ex(t)=0$, the left-hand side of Eq. (23) in [2] implies

$$x(t)^{\mathrm{T}}\Big[E^{\mathrm{T}}PA + A^{\mathrm{T}}PE - \frac{1}{\varepsilon}E^{\mathrm{T}}PBR^{-1}B^{\mathrm{T}}PE + \frac{1}{\varepsilon}E^{\mathrm{T}}PB_{\mathrm{d}}R^{-1}B_{\mathrm{d}}^{\mathrm{T}}PE + E^{\mathrm{T}}PA_{\mathrm{d}}A_{\mathrm{d}}^{\mathrm{T}}PE + \frac{1}{\gamma}C^{\mathrm{T}}C + \frac{1}{\gamma}E^{\mathrm{T}}PDD^{\mathrm{T}}PE + I_n\Big]x(t) = x(t)^{\mathrm{T}}\Big[\frac{1}{\gamma}C^{\mathrm{T}}C + I_n\Big]x(t). \tag{4}$$