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Limit cycle analysis of nonlinear sampled-data systems by gain–phase margin approach

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Abstract

This work analyzes the limit cycle phenomena of nonlinear sampled-data systems by applying the methods of gain–phase margin testing, the *M*-locus and the parameter plane. First, a sampled-data control system with nonlinear elements is linearized by the classical method of describing functions. The stability of the equivalent linearized system is then analyzed using the stability equations and the parameter plane method, with adjustable parameters. After the gain–phase margin tester has been added to the forward open-loop system, exactly how the gain–phase margin and the characteristics of the limit cycle are related can be elicited by determining the intersections of the *M*-locus and the constant gain and phase boundaries. A concise method is presented to solve this problem. The minimum gain–phase margin of the nonlinear sampled-data system at which a limit cycle can occur is investigated. This work indicates that the procedure can be easily extended to analyze the limit cycles of a sampled-data system from a continuous-data system cases considered in the literature. Finally, a sampled-data system with multiple nonlinearities is illustrated to verify the validity of the procedure.

Keywords: Limit cycle; Gain–phase margin; Sampled-data; Describing function

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1. Introduction

Accurately predicting the limit cycles of nonlinear control systems has been relevant to many industrial applications using describing function method. Hydraulic and robotic control systems with friction have been addressed in recent years [1,2]. In [3], a car steering control with an actuator rate limit was considered. In addition, Amato et al. analyzed pilot-in-the-loop oscillations due to positional and rate saturations [4].

Although the controller of a linearized model can be designed by any classical or modern method, the limit cycle characteristics of the designed system must be thoroughly analyzed, since the system is likely to exhibit undesirable limit cycles caused by nonlinearities. Uncertain parameters in a linear control system can be robustly analyzed by the parameter plane method or the parameter space method [5–8]. The *M*-locus method can also be implemented to represent the describing function of nonlinearities in the parameter plane or parameter space to determine the characteristics of limit cycles [9]. The above methods can be used to decide carefully the safe range of system parameters to avoid the generation of limit cycles.

In the frequency-domain approach, gain margin (GM) and phase margin (PM) are two important indices for analyzing and designing practical control systems. Methods of analyzing the gain–phase margin in a linear continuous-data system with adjustable parameters have recently been developed [10]. This approach has been extended to analyze a nuclear reactor system with various transport lags [11] and a proportional navigation sampled-data control system [12]. Thereafter, the prediction of limit cycles in some nonlinear control systems, such as a reactor system and a low-flying vehicle, was analyzed in [13–15]. The authors of the current investigation also addressed the gain–phase margin analysis of pilot-induced oscillations for predicting limit cycles [16].

A systematic strategy, similar to [16], is first presented to predict the limit cycles caused by the effects of parameter variations and hard nonlinearities and, in doing so, to extend the above results to the sampled-data control systems. A simple method for evaluating the gain–phase margins and the *M*-locus in the parameter plane is also proposed for analyzing stability by inserting a gain–phase margin tester into the forward open-loop of a linearized sampled-data control system. Importantly, the developed approach is very easy to implement and can provide more information on limit cycles. Finally, two examples of nonlinear sampled-data system are given to confirm the proposed design procedures.

The rest of this paper is organized as follows. Section 2 presents the basic approach. Section 3 presents the first example of an aircraft pitch control system, including one nonlinear element provided to demonstrate the design procedures. Section 4 extends the approach to analyze a sampled-data control system with multiple nonlinearities. Finally, Section 5 draws conclusions.

2. Conventional approach

This section addresses the conventional approach in predicting the limit cycles of nonlinear sampled-data control systems.