ISA Transactions 44 (2005) 145–163

ISA TRANSACTIONS®

Design and analysis of controllers for a double inverted pendulum

Henrik Niemann,[a,*] Jesper Kildegaard Poulsen[b,†]

[a]*Ørsted-DTU, Automation, Technical University of Denmark, Building 326, DK-2800 Lyngby, Denmark*
[b]*Intentia Danmark A/S, Borupvang 5 D-E, DK-2750 Ballerup, Denmark*

(Received 25 November 2002; accepted 7 May 2004)

Abstract

A physical control problem is studied with the $\mathcal{H}_\infty$ and the μ methodology. The issues of modeling, uncertainty modeling, performance specification, controller design, and laboratory implementation are discussed. The laboratory experiment is a double inverted pendulum placed on a cart. The limitations in the system with respect to performance are the limitation in the control signal and the limitation of the movement of the cart. It is shown how these performance limitations will effect the design of $\mathcal{H}_\infty$ and μ controllers for the system. © 2005 ISA—The Instrumentation, Systems, and Automation Society.

Keywords: Modeling; $\mathcal{H}_\infty$ controller design; μ synthesis; Simulation; Controller implementation; Laboratory experiments

1. Introduction

The pendulum system is one of the classical examples used in connection with feedback control. The inverted single pendulum is a standard example in many text books dealing with classical as well as modern control. The reason is that the system is quite simple, nonlinear, and unstable. In connection with the classical control, the single inverted pendulum system has among other things been used to show that the system cannot be stabilized by using just a P controller. In spite of the fact that the system is unstable, the design of stabilizing controllers for the system can be done reasonably easy. However, this is not the case when considering the quite more complicated double inverted pendulum system. It is more complicated to design/tune stabilizing controllers for the system. Therefore more advanced controller architectures and advanced design methods need to be applied. This involved different types of model based controllers designed by using, e.g., $\mathcal{H}_2$ based methods, $\mathcal{H}_\infty$ based methods, and μ based methods.

A benchmark problem for robust controller design derived by Wei and Bernstein [1] has been described in a special issue in AIAA Journal of Guidance, Control, and Dynamic, Vol. 15, No. 5, 1992. The benchmark problem is a linear two-mass-spring system with uncertain masses and spring constant. Compared with the double inverted pendulum system, the linear benchmark problem is less complicated, but still a quite challenged design problem. A number of different design approaches has been applied on the linear benchmark problem, among them also $\mathcal{H}_\infty$ controller design and μ synthesis, see Refs. [2–7].

The main problem in the design of stabilizing controllers for the double inverted pendulum system is the tradeoff between robust stability and performance. This tradeoff is limited and there is not much space for reduction of the robustness to increase the performance of the system. The reason is the nonlinearities in the system together with the limitations/saturations in the system. The

*E-mail address: hhn@oersted.dtu.dk
†E-mail address: jesper.k.poulsen@intentia.dk

Nomenclature

θ_1	angle between vertical and the lower arm
$\dot{\theta}_1$	angular velocity related to θ_1
θ_2	angle between vertical and the upper arm
$\dot{\theta}_2$	angular velocity related to θ_2
x_c	cart position
$\dot{x}_c$	velocity of the cart
i	motor current
U	reference voltage to motor/tacho system, control signal
r_c	cart position reference
M_{d_1}	torque disturbance on the joint on the lower arm
M_{d_2}	torque disturbance on the joint on the upper arm
M_{dm}	torque disturbance on the motor
n_1	noise signal in measuring θ_1
n_3	noise signal in measuring $\theta_3 = \theta_1 - \theta_2$
n_x	noise signal in the measuring of the cart position x_c
e_c	cart position error $r_c - x_c$

mass of cart	m	0.81 kg
length of track	l_t	1.34 m
mass of lower arm	m_1	0.548 kg
length of lower arm	l_1	0.535 m
length of lower arm from bottom to center of mass	l_{1cm}	0.355 m
inertia of lower arm around the lower joint	I_1	$2.678e^{-2}$ kg m^2
mass of upper arm	m_2	0.21 kg
length of upper arm	l_2	0.512 m
length of upper arm from bottom to center of mass	l_{2cm}	0.12 m
inertia of upper arm around the lower joint	I_2	$5.217e^{-3}$ kg m^2

limitations in the system are, e.g., maximal power to the motor (maximal acceleration of the cart), and maximal length of the track, to mention the two most important limitations. In spite of this limited tradeoff between robustness and performance of the system, it is possible to design controllers that can handle this tradeoff in a systematic way. Design methods such as, e.g., $\mathcal{H}_2$, $\mathcal{H}_\infty$, and μ based methods can be applied for handling this tradeoff in a systematic way.

This paper describes a complete design procedure for the design of advanced stabilizing controllers for the double inverted pendulum system together with an implementation of the controllers on a laboratory system. This leads to the following items that will be considered:

- system modeling,
- system analysis,
- uncertainty modeling,
- design problem formulation,
- controller design,
- analysis of the closed-loop system,
- implementation of controllers on a microcontroller,
- validation of the closed-loop system on the laboratory system.

Due to the lack of space, the first three items will only be considered briefly. A more detailed description of these parts can be found in Ref. [8]. This paper is organized as follows: In Section 2,

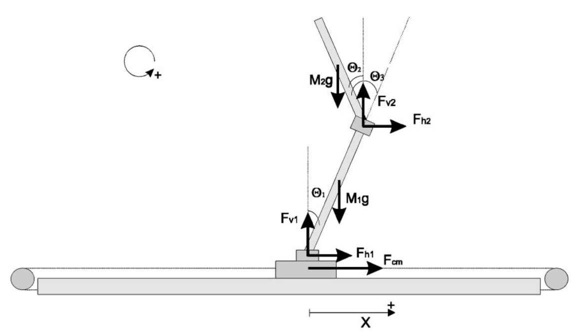

Fig. 1. Principal diagram of the double inverted pendulum system.