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The performance of unit root tests under level-dependent heteroskedasticity

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Abstract

Several financial variables exhibit level-dependent conditional heteroskedasticity. This may cause severe distortions in conventional unit root tests. Given the absence of theoretical results, we conduct Monte Carlo investigation to assess the performance of the standard Dickey–Fuller tests, a nonparametric alternative, and a heteroskedastic-robust extension of the Dickey–Fuller *t*-test. While these procedures have approximately correct size, we find strong distortions in the power of the standard Dickey–Fuller tests.

Keywords: Interest rates; DF test; CKLS; Double-autoregressive process; Nonparametric test

JEL classification: C12; C15; C52; E43

1. Introduction

In this paper, we investigate the properties of several unit root tests when applied to finite-sized samples of data generated from a process with level-dependent volatility. This issue is particularly important for applications on interest rates time-series, since there are economic reasons to assume that

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these variables follow a mean-reverting stochastic process with variability related to their levels (see, inter alia Cox et al., 1985; Chan et al., 1992). Empirical evidence widely supports this particular form of conditional heteroskedasticity. While the mean-reverting hypothesis is usually tested against the null of a unit root, level-dependent heteroskedasticity raises a number of econometric implications. Fairly standard methods, such as the Dickey–Fuller test (DF henceforth), may no longer be valid, since several standard assumptions (e.g. errors with finite variance) do not hold in this context.

This relevant issue has not been dealt with in the nonstationary literature, which has almost exclusively focused on the effects of GARCH errors (see, for instance, Kim and Schmidt, 1997). We provide insight through Monte Carlo simulations on the size and power of the standard DF tests (t-ratio and F-test), the heteroskedasticity-robust DF t-ratio based on White's corrected standard errors suggested by Haldrup (1994), and a nonparametric test proposed by Breitung (2002). Even though some key assumptions are not fulfilled, we find that these test procedures have approximately correct empirical size. However, in the case of the standard DF tests, level-dependent heteroskedasticity can lead to strong distortions in power. Thus, the nonparametric or the heteroskedasticity-robust parametric test procedures may represent a better alternative in this context.

2. Testing the null of a unit root

Consider the discrete-time version of the well-known model in Chan et al. (1992),

$$r_t = \mu + \phi r_{t-1} + \varepsilon_t;\ \ \varepsilon_t = \sigma r_{t-1}^{\gamma}\eta_t;\ \ \eta_t\,|\,I_{t-1} \sim \text{iid}(0,1);\ \ \gamma > 0. \tag{1}$$

This model nests a large number of one-factor models for interest rates. It is also a generalization of the homoskedastic AR(1) process and of so-called double-autoregressive models (Ling, 2004), for which γ is constrained to be equal to unity. Due to its generality, (1) will serve as benchmark for our analysis.

The null hypothesis of a unit root, i.e. H_0: $\phi = 1$, has strong implications on a process like (1). First, r_t finds an absorbing barrier at zero which is asymptotically attainable, so that r_t degenerates at zero asymptotically. Second, the error term is defined on an integrated process, and clearly $\lim_{t\infty} E\left(\varepsilon_t^2\right) = \infty$. Finally, r_t may be nonstationary even under the alternative hypothesis if $\gamma > 1$, as shown in Broze et al. (1995). These features largely interfere with asymptotic analysis whereby the derivation of the limit distribution under H_0: $\phi = 1$ remains a challenging and unsolved problem.

Unit root tests based on the assumption $E(\varepsilon_t^2) < \infty$ are likely to suffer from severe size distortions when errors are drawn from heavy-tailed distributions, or if a second moment does not exist. We first address the implications on the size and power of the DF tests. This procedure is popular because of its simplicity, and is frequently applied in empirical research on interest rates. The standard DF test statistics are based on OLS estimates from an auxiliary regression such as,

$$\Delta r_t = \mu + \alpha r_{t-1} + v_t \tag{2}$$

where $E(v_t^2) < \infty$ is assumed. Clearly, the basic DF regression model assumes $\gamma = 0$ in (1). The relevant test statistics are a one-sided t-ratio for the significance of α and an F-test for the joint hypothesis $\mu = \alpha = 0$. If needed, (2) may be augmented with lags of the dependent variable to account for short-run dynamics. Alternatively, the DF t-ratio may be computed using White's corrected standard errors [DF-W] if heteroskedasticity is suspected (Haldrup, 1994; Kim and Schmidt, 1993).