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Linear-risk-tolerant, invariant risk preferences

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Abstract

This note identifies the class of preferences which simultaneously satisfy invariance, two-fund portfolio separation, and linear risk tolerance. It also considers the implications for asset demand and asset pricing of this class of preferences.

Keywords: Uncertainty; Preferences; Asset demand

JEL classification: D81; G11; G12

Quiggin and Chambers (2004) have introduced the notion of invariant preferences. These preferences generalize both the class of constant risk averse preferences (Safra and Segal, 1998; Quiggin and Chambers, 1998; Chambers and Quiggin, 2002) and mean-standard deviation preferences. In particular, Quiggin and Chambers (2004) show that preferences are invariant, if and only if the certainty equivalent, e, assumes the general form:

$$e(\boldsymbol{y}) = \phi\big(\mu_{\hat{\pi}}(\boldsymbol{y}), \rho\big(\boldsymbol{y} - \mu_{\hat{\pi}}(y)\boldsymbol{1}\big)\big),$$

where $\boldsymbol{y}$ is a vector of state-contingent incomes, ϕ is a, real-valued function increasing in its first argument and decreasing in its, second, $\hat{\pi}$ is a given probability vector, $\mu\hat{\pi}(\boldsymbol{y})$ is the mean of the state-contingent income vector, evaluated with respect to $\hat{\pi}$, $\boldsymbol{1}$ is a vector of ones, and ρ is a nonnegative,

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lower semi-continuous, positively linearly, homogeneous, and subadditive function. ρ, thus, generalizes the standard deviation so that mean-standard deviation preferences are invariant.

Quiggin and Chambers (2004) show that the only invariant expected-utility functionals are those associated with a quadratic ex post utility function. This class of preferences has some very unattractive properties when regarded as preferences over wealth, but they also satisfy the conditions for two-fund portfolio separation and exhibit linear risk tolerance over a restricted domain. Invariant preferences always satisfy a form of two-fund portfolio separation in the presence of a riskless asset (Quiggin and Chambers, 2004). This note identifies the class of preferences which simultaneously satisfy invariance, two-fund portfolio separation, and linear risk tolerance to determine if there exist meaningful classes of preferences which inherit much of the quadratic family's theoretical and empirical tractability but do not necessarily inherit its more unattractive properties when regarded as preferences over wealth.

Our analysis relies on the dual treatment of risk-averse preferences developed by Chambers and Quiggin (2002). In what follows, we first introduce some notation and basic concepts. Then we briefly discuss the translation and expected-value functions and then use these concepts to deduce necessary and sufficient conditions for individual preferences to be both invariant and linear-risk-tolerant. Finally, we consider implications for asset demand and asset pricing.

1. Notation and basic concepts

We consider preferences over random variables represented as mappings from a state space Ω to a convex outcome space $Y \subseteq \Re$. Ω is a finite set $\{1,\ldots,S\}$, and the space of random variables is, thus, $Y^{\Omega} \subseteq \Re^{\Omega}$. The unit vector is denoted $\mathbf{1}=(1,1,\ldots,1)$, and $\mathcal{P} \subset \Re^S_+$ denotes the probability simplex. The vector $\hat{\pi} \in \mathcal{P}$ is taken to represent known (subjective or objective) probabilities over the state space.

Preferences over state-contingent incomes are given by the certainty equivalent $e(\mathbf{y})$, which is continuous, nondecreasing, and quasi-concave in $\mathbf{y}$. Quasi-concavity ensures that the least-as-good sets of the preference mapping

$$V(e) = \{\mathbf{y} : e(\mathbf{y}) \geq e\}$$

are convex, and that the individual is risk averse in the sense of Yaari (1969).

2. The translation function and the expected-value function

The *translation function*, $B : \Re \times Y^S \rightarrow \Re$, is defined:

$$B(e,\mathbf{y}) = \max\{\beta \in \Re : \mathbf{y} - \beta\mathbf{1} \in V(e)\}$$

if $\mathbf{y} - \beta\mathbf{1} \in V(e)$ for some β and $-\infty$ otherwise (Blackorby and Donaldson, 1980; Luenberger, 1992).[1] The properties of $B(e,\mathbf{y})$ are well known (Blackorby and Donaldson, 1980; Luenberger, 1992; Chambers et al., 1996; Chambers and Quiggin, 2002). Most importantly for our purposes, it is nonincreasing in e and nondecreasing and concave in $\mathbf{y}$.

[1] The translation function is a special case of the benefit function defined by Luenberger (1992).