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# A tractable model of precautionary saving in continuous time

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**Abstract**

This letter derives an analytical solution to the following saving problem. An individual chooses the path of consumption that maximises a time-separable von Neumann–Morgenstern utility function, subject to a standard intertemporal budget constraint. Labour income follows a Poisson process. The consumer is both 'prudent' and 'impatient', and accumulates financial wealth as a buffer against the risk of income loss.
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## 1. Introduction

Studies on optimal consumption in risky environments and imperfect markets provide a framework that can match many of the important features of the empirical data on consumption saving and data; see Carroll (2001) and references therein. A difficulty commonly encountered is the mathematical sophistication associated with realistic specifications of the models. We present a tractable model of precautionary saving in continuous time. Labour income is non-diversifiable. Preferences exhibit constant relative risk aversion in consumption (CRRA). Optimal consumption is characterised by concavity with respect to wealth.

The key assumption is that individuals face, throughout their working life, a probability $\mu$ of losing their job. Thus their expected working life is $1/\mu$. The parameter $\mu$ may be chosen anywhere between

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zero and infinity. In the limit, as $\mu$ goes to zero, individuals face no labour income risk and the precautionary motive disappears. The problem is tractable because once the uncertainty is realized the consumption function may be obtained in closed form. The main advantage of our analytical solution is the use of the CRRA utility and the use of a standard phase diagram analysis. The solution is developed with the familiar tool of optimal control theory in continuous time. Our approach is attractive for modelling the uncertainty associated with rare, large, permanent income losses, such as the risk of serious injury or compulsory retirement. However, it does not capture the more moderate uncertain fluctuations of labour income in the course of an individual's working life. The only source of uncertainty is about the timing of the income loss, and not about its magnitude (this simplification could be relaxed) or about its persistence (this is the key assumption). In this setting, it is possible to give an analytical characterisation of optimal consumption. Individuals engage in buffer-stock saving behaviour, accumulating financial wealth to smooth consumption in the event of an income loss.

## 2. An optimal consumption problem

Individuals select a flow of consumption $C_t$ to maximise

$$E_t \int_t^{\infty} \mathrm{e}^{-\rho(s-t)} u(C_s)\mathrm{d}s$$

where $E_t$ is the conditional expectation operator, and $\rho$ is the rate of pure time preference. The utility index is additively separable in time, and discounted at a constant and positive rate. The utility function is isoelastic,

$$u(C) = \frac{C^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}},$$

where $\sigma > 0$ is the elasticity of intertemporal substitution. The individual faces a standard budget constraint,

$$\dot{A}_t = r_t A_t + \tilde{Y}_t - C_t,$$

where $\tilde{Y}_t$ is random labour income, $A_t$ is financial wealth, and $r_t$ is the rate of interest. The event that labour income, measured in effective units, drops permanently from $w_t$ to 0 follows a Poisson process with arrival rate $\mu$, where $w_t$ is the wage rate received if working:

$$\tilde{Y}_t = \begin{cases} w_t L_t & \text{with probability } 1-\mu \mathrm{d}t \\ 0 & \text{with probability } \mu \mathrm{d}t, \end{cases}$$

where $L_t$ denotes the number of effective units. Let $L_t$ grow at a constant (exogenous) rate $\dot{L}_t / L_t = g$.

The deterministic problem faced by the individual whose non-financial income has forever dropped to 0 may be solved independently of the stochastic problem faced by the individual whose labour income is the random variable $\tilde{Y}_t$. It is thus possible to solve the full problem by 'backward induction', solving first for the deterministic problem and secondly for the stochastic problem. Let superscripts index the individual's current state; thus $C_t^{\mathrm{e}}$ stands for consumption when employed, and $C_t^{\mathrm{u}}$ if unemployed.

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