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Estimating and identifying vector autoregressions under diagonality and block exogeneity restrictions

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Abstract

I show how to estimate and identify a large-scale vector autoregression (VAR) when the variables in a subset are mutually independent, conditional on common factors and when the conditioning variables are independent of the former subset. The approach is useful when using VARs to estimate the responses of a large cross-section of variables to aggregate shocks.

Keywords: VAR; Impulse response function; Estimation; Identification

JEL classification: C32

1. Introduction

Vector autoregressions (VARs) are effective tools for analyzing the dynamics of a stochastic system and for making economic inference. However, data constraints typically mean that large dimensional systems suffer from insufficient degrees of freedom and thus lack robustness. This note shows how to estimate and identify large-scale structural VARs when (a) the mutual correlation among a subset of the variables in the system is due solely to joint dependence on a separate subset variables and (b) the latter subset is independent of the former. The assumption of mutual independence conditional on a set of

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'common factors' imposes a diagonal structure on part of the VAR, while the second assumption implies that the common factors are block exogenous. I show that least square methods are efficient and that identification of the structural dynamic (impulse) responses of the model requires restrictions only on the subset of common variables. The approach will be most useful when it is desirable to use VAR models to examine the dynamic responses of a large cross-section of variables, such as industry-level prices or output, to aggregate shocks.

2. The model

Let $z_t = \begin{pmatrix} z_{1t} \\ z_{2t} \end{pmatrix}$ be an n-dimensional vector stochastic process, where z_{1t} is $n_1 \times 1$, z_{2t} is $n_2 \times 1$ and $n = n_1 + n_2$. Assume that this process is generated by the linear dynamic model:

$$A_0 z_t = A_1 z_{t-1} + \ldots + A_p z_{t-p} + u_t, \tag{1}$$

where $u_t = \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix}$ is a white noise vector process normalized so that $E u_t u_t' = I$ and A_i, $i = 0, \ldots, p$, is $n \times n$. The corresponding reduced form of this structural model is

$$z_t = A_0^{-1} A_1 z_{t-1} + \ldots + A_0^{-1} A_p z_{t-p} + A_0^{-1} u_t$$

$$= B_1 z_{t-1} + \ldots + B_p z_{t-p} + \epsilon_t, \qquad E \epsilon_t \epsilon_t' \equiv \Omega. \tag{2}$$

The system in Eq. (2) is the VAR representation of the structural model in Eq. (1). The moving average representation of the structural model is

$$z_t = \left(A_0 - A_1 L - \ldots - A_p L^p\right)^{-1} u_t \tag{3a}$$

$$= \left(D_0 + D_1 L + D_2 L^2 + \ldots\right) u_t \tag{3b}$$

$$= D(L) u_t. \tag{3c}$$

Likewise, the reduced form moving average is

$$z_t = \left(I - B_1 L - \ldots - B_p L^p\right)^{-1} \epsilon_t \tag{4a}$$

$$= \left(I + C_1 L + C_2 L^2 + \ldots\right) \epsilon_t \tag{4b}$$

$$= C(L) \epsilon_t. \tag{4c}$$

The objective is to identify the economic structure in Eqs. (3a–c) from the moving average in Eqs. (4a–c), which is directly determined by estimating the coefficients in Eq. (2). In particular, the parameters of interest are the structural dynamic multipliers or impulse response functions: $\frac{\partial z_{t+k}}{\partial u_t} = D_k$. The empirical strategy entails estimating $B(L)$ and Ω from Eq. (2), then imposing restrictions on the structure to identify the parameters of interest from these estimates.