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Games and Economic Behavior 53 (2005) 43–58
GAMES and Economic Behavior
www.elsevier.com/locate/geb

Distributing awards efficiently: More on King Solomon's problem

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Received 10 July 2002

Available online 10 December 2004

Abstract

We consider a multi-awards generalization of King Solomon's problem: k identical prizes should be distributed, without any monetary transfers in equilibrium, among $n > k$ agents, with the top k valuation agents receiving the awards. Glazer and Ma [1989. Efficient allocation of a prize—King Solomon's dilemma. Games Econ. Behav. 1, 222–233] analyzed the single-prize version of this problem assuming complete information amongst the agents. We show that with more than two agents the mechanism of Glazer and Ma admits inefficient equilibria and thus fails to solve Solomon's problem. So, first we modify their mechanism to rule out inefficient equilibria and implement efficient prize allocation for arbitrary number of agents. Then it is shown that the multiple-awards case can be implemented in *subgame perfect equilibrium* by a repeated application of the modified single-award mechanism. Finally, we relax the informational assumption and show, using a generalized version of Olszewski's [2003. A simple and general solution to King Solomon's problem. Games Econ. Behav. 42, 315–318] mechanism, that multi-awards problem can also be implemented by iterative elimination of weakly dominated strategies.

JEL classification: D78

Keywords: Solomon's problem; Prizes; Implementation

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0899-8256/$ – see front matter
doi:10.1016/j.geb.2004.09.002

1. Introduction

Glazer and Ma (1989) had formally addressed King Solomon's dilemma—the problem of giving a baby to the baby's true mother with two women both claiming to be the true mother. This problem, as Glazer and Ma had noted, is generically equivalent to awarding an indivisible prize to one of several agents who valued the prize most. While the agents knew each others' valuations, the planner (or King Solomon) had no such information. Glazer and Ma constructed extensive form mechanisms implementing the award rule in subgame perfect equilibrium (SPE) without any monetary transfers in equilibrium.[1]

More recently, Perry and Reny (1999) have relaxed the complete information assumption of Glazer and Ma: in the case of two contenders as in King Solomon's problem, each agent knows her own value and each agent knows which of them has the higher value; however, neither agent knows the precise value of the other agent.[2] The authors suggest a variant of the second-price sealed-bid all-pay auction that implements the efficient allocation in iteratively undominated strategies. Later, Olszewski (2003) has constructed a 'simpler' mechanism that requires only two rounds of elimination (of weakly dominated strategies) as opposed to Perry and Reny's (1999) four rounds of elimination.

Our primary objective is to generalize King Solomon's problem in a different direction, assuming complete information. We consider the problem of distributing k identical and indivisible prizes among n agents where $k < n$.[3] The objective of the planner is to give the prizes (in equilibrium) to the *top k valuation agents* at zero monetary costs to the planner and the agents.[4] The top k valuation agents need not be unique: corresponding to a decreasing order of valuations $u_{(1)} \geqslant u_{(2)} \geqslant \cdots \geqslant u_{(k)} \geqslant \cdots \geqslant u_{(n)}$, there can be more than one ordering of the agent indexes with more than one agent tied at the valuation $u_{(k)}$. Thus, the agents who receive the prizes must have valuations among the top k ranks in at least one of the corresponding ordering of agent indexes.

To generalize the result of Glazer and Ma (1989) in the case of multiple awards requires a further analysis of their single-prize mechanism for *three or more* agents; they formally prove the implementation result for the two-agents problem only and in the appendix they outline a more elaborate mechanism claiming that it implements efficient allocations when there are at least three agents. It turns out that this latter claim of Glazer and Ma is *not always* true. There are two problems with their mechanism, one trivial and a second problem more substantial due to ties in agent valuations. As a result, the Glazer–Ma mechanism results in multiple equilibria involving inefficient allocations of the prize. So, first we modify the Glazer–Ma mechanism to complete the task of solving King Solomon's problem for

[1] Moore (1992) suggested a mechanism implementing Solomon's choice function in undominated Nash equilibrium, but criticized the mechanism for its use of an 'integer game' construction. He also described an extensive form mechanism, similar to Glazer and Ma's construction, for implementation in SPE.

[2] In the case of more than two contenders, also considered by Perry and Reny, each agent knows her own value and the identity of the highest value agent, and the highest value agent knows that her value is strictly higher than all other agents' values. In fact, only the highest value agent might know that she has the highest value.

[3] $k = 1$ is the problem considered by Glazer and Ma.

[4] Bag (1996) considered a related prize distribution problem—the problem of dividing a given amount of *divisible* resources among n agents. In the current paper, the prizes are *indivisible*.