Available online at www.sciencedirect.com
SCIENCE DIRECT®

Games and Economic Behavior 51 (2005) 420–444

GAMES and Economic Behavior

www.elsevier.com/locate/geb

Strongly stable networks

Matthew O. Jackson [a,*], Anne van den Nouweland [b]

[a] *HSS 228-77, California Institute of Technology, Pasadena, CA 91125, USA*
[b] *Department of Economics, University of Oregon, Eugene, OR 97403-1285, USA*

Received 3 June 2003

Available online 10 December 2004

Abstract

We analyze the formation of networks among individuals. In particular, we examine the existence of networks that are stable against changes in links by any coalition of individuals. We show that to investigate the existence of such strongly stable networks one can restrict focus on a component-wise egalitarian allocation of value. We show that when such strongly stable networks exist they coincide with the set of efficient networks (those maximizing the total productive value). We show that the existence of strongly stable networks is equivalent to core existence in a derived cooperative game and use that result to characterize the class of value functions for which there exist strongly stable networks via a "top convexity" condition on the value function on networks. We also consider a variation on strong stability where players can make side payments, and examine situations where value functions may be non-anonymous—depending on player labels.

JEL classification: D85; A14; C71; C72

Keywords: Networks; Network formation; Strong stability; Allocation rules; Core

* Corresponding author.
E-mail addresses: jacksonm@hss.caltech.edu (M.O. Jackson), annev@uoregon.edu (A. van den Nouweland).

0899-8256/$ – see front matter
doi:10.1016/j.geb.2004.08.004

1. Introduction

The importance of networks in a variety of social and economic settings is well-documented. Applications range from social networks such as friendships to more directly economically motivated ones such as trading alliances, decentralized market relationships, research partnerships, etc. Given that network relationships matter, it is important to understand which networks are likely to form and how this depends on the structure of the setting. In particular, there has been a good deal of recent research into understanding how networks form among a group of individuals (people, firms, etc.) who have the discretion to choose with whom they interact.[1]

In this paper, we continue that line of research through a careful study of the existence and properties of strongly stable networks: those networks which are stable against changes in links by any coalition of individuals. Strongly stable networks are those which are supported by strong Nash equilibria of an appropriate game of network formation.

There are many reasons for studying a strong notion of stability based on coalitional considerations. In network formation, individual or pairwise based solution concepts such as Nash equilibrium and pairwise stability (see Jackson and Wolinsky, 1996) often lead to many stable networks, so that they provide broad predictions. In some contexts this already narrows things, but in other contexts it may leave us with a large set of networks. Moreover, these networks may have very different properties and then additional considerations may help us to sort among them to produce narrower and more accurate predictions of network formation. (See Examples 1 and 2, in Section 2.1, for illustrations.) In addition, in many contexts, there will naturally be communication among individuals that may allow a number of them to coordinate their choices of links. As such, we study strongly stable networks as a natural way for making tighter predictions using coalitional considerations. One can think of a notion such as pairwise stability as a weak stability concept which is essentially a necessary (and some times too weak) condition for stability, while strong stability is a sufficient (and some times too strong) condition for stability.

Strong stability of networks is a very demanding property, as it means that no set of players could benefit through any rearranging of the links that they are involved with (including those linking them to players outside the coalition). As such, we expect there to be contexts where such networks will not exist. However, strongly stable networks still exist in a number of natural settings, including some that pop up in the literature as examples of network situations. In situations where strongly stable networks exist they are quite compelling, in the sense that once formed such networks are essentially impossible to destabilize, as there is no possible reorganization that would be improving for all of the players whose consent is needed.

Another reason for examining the existence of strongly stable networks, beyond their compelling stability properties, is that such networks exhibit additional properties. For instance, as we shall show, if a network is strongly stable and has more than one component, then value must be allocated equally among members of each component, and in fact the per capita value must be equal across components. This is a very strong equity property.

[1] For bibliographies on network study generally and network formation in particular, we refer the reader to Slikker and van den Nouweland (2001a), Dutta and Jackson (2003), and Jackson (2004).