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Core convergence with asymmetric information

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Abstract

We analyze the ex ante incentive compatible core for replicated private information economies. We show that any allocation in the core when the economy is replicated sufficiently often is approximately Walrasian for the associated Arrow–Debreu economy.

JEL classification: C7; C71; D5

1. Introduction

The Debreu–Scarf core convergence theorem (Debreu and Scarf, 1972) is a fundamental result in general equilibrium analysis: under suitable convexity assumptions, any allocation that is not an Arrow–Debreu equilibrium will be blocked in a sufficiently large replica economy. The theorem suggests why trade among many agents will lead to a system of prices that agents take as given when minimal assumptions on the stability of allocations are imposed.

Our aim in this paper is to prove a core convergence theorem for exchange economies in the presence of asymmetric information. The particular manner in which we model the asymmetry of information follows the development in McLean and Postlewaite (2002a, 2003). Agents' utility functions will depend on an underlying but unobserved state

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of nature θ, and each agent will receive a private signal that is correlated with the state of nature. A replication of this initial economy consists of a set of agents whose utility functions and initial endowments are the same as those in the underlying initial economy, but whose private signals are independent conditional on θ. No agent's information is redundant in this replication procedure: regardless of the number of replications, each agent still has information that can not be inferred from the aggregate information of other agents.

If the state θ on which the agents' utilities depend were observable prior to consumption, then agents could exchange state contingent goods prior to the realization of θ. However, in our model θ is *not* observable; all information about the realization of θ is embodied in the vector of agents' types. Consequently, agents can trade bundles contingent on the realized vector of types but not contingent on θ. Since agents' types are independent conditional on θ, by the law of large numbers, the vector of agents' types will (with high probability) provide a highly accurate prediction of the realized θ when the number of agents gets large. Hence, in this case one might hope that allocations contingent on the agents' information might approximate desirable allocations contingent on θ.

The main difficulty in formalizing this idea arises from the observation that allocations contingent on agents' information may not be incentive compatible. There are several core concepts one might employ. In this paper, we study the *ex ante incentive compatible core*, in which decisions are made at the ex ante stage and incentive constraints are taken into account. Our main theorem shows that if an asymmetric information exchange economy is replicated sufficiently many times, any ex ante incentive compatible core allocation is approximately competitive in the sense that, for almost all agents, the utility from the core allocation is close to the utility they receive at a certain complete information competitive equilibrium allocation. Thus, asymmetric information economies asymptotically behave like complete information economies as far as core-type stability is concerned.

Several complications arise in the analysis of the core when an economy with asymmetrically informed agents is replicated. First, while the core with complete information is nonempty under quite general circumstances, Vohra (1999) and Forges et al. (2002) show that the ex ante incentive compatible core may be empty in well-behaved pure exchange economies. However, in McLean and Postlewaite (2003), we showed that when agents are sufficiently "informationally small," the ex ante incentive compatible core is nonempty. Further, they show that the informational size of agents will converge to zero when asymmetric information economies are replicated in a natural manner, and, consequently, the ex ante incentive compatible ε-core will be nonempty after a suitable number of replications.

The second complication in investigating the ex ante incentive compatible core with replication is technical. A key step in the proof of the Debreu–Scarf theorem is the argument that any core allocation must satisfy an "equal treatment" property. The equal treatment property states that all replicas of a given type must receive the same bundle in any core allocation. This property greatly simplifies the analysis since the dimensionality of the space of allocations goes to infinity when the number of replications goes to infinity, but attention can be restricted to allocations that are feasible for the initial economy.