Available online at www.sciencedirect.com
SCIENCE DIRECT®
Insurance: Mathematics and Economics 36 (2005) 37–55
INSURANCE MATHEMATICS & ECONOMICS
www.elsevier.com/locate/econbase

Pricing optional group term insurance: a new approach using reservation prices

Colin M. Ramsay

University of Nebraska-Lincoln, Finance Department, Lincoln, NE 68588-0426, USA

Received March 2004; received in revised form September 2004; accepted 28 September 2004

Abstract

Consider an employer who, through an insurer, provides optional group term life insurance to a group of employees. The employees are assumed to have mortality following a mixture mortality model where they have different mortality rates belonging to a common probability distribution. To reduce the effects of possible adverse selection, the insurer sets a maximum acceptable mortality level (q^M). The insurer then uses a costly medical underwriting/exam to determine each applicant's mortality level, q. If $q > q^M$ the employee is refused insurance otherwise insurance is granted. Each employee is assumed to have a reservation price for term insurance. Economic theory is used to determine the employees' inverse aggregate demand function. This demand function is then used to determine the mortality cut-off level and premium that maximize the insurer's expected profits. First order conditions and several necessary conditions for profit maximization are given.

JEL classification: G22; D81; D82

Subj. Class.: IB12; IE41; IM30

Keywords: Underwriting; Medical exam; Mortality differential; Mixture mortality; Term insurance; Adverse selection; Mortality cut-off; Profit maximization; Lagrange expansion

1. An overview of traditional insurance pricing

1.1. Non-life insurance

Consider a group of N homogeneous risk averse persons where the kth person has a potential non-life loss of $X_k \geq 0, k = 1, 2, \ldots, N$, in the current period. Each person desires full insurance to cover the entire possible loss. Assuming losses are independent and identically distributed, the traditional actuarial approach is to charge each

E-mail address: cramsay@unlnotes.unl.edu.

0167-6687/$ – see front matter © 2004 Elsevier B.V. All rights reserved.
doi:10.1016/j.insmatheco.2004.09.003

insured a gross premium G where

$$G = \mathbb{E}[X_k] + \mathrm{EXP}_k + \mathrm{PFT}_k + R_k,$$

where EXP_k and PFT_k are the expense and profit loads, respectively, which reflect the insurer's expected expenses and profit per insured. The term R_k represents the risk (or contingency) load that is added to cover adverse deviation (i.e., higher than expected losses), and it typically decreases as N increases.

The risk load is often determined by the premium calculation principle used. There are several popular premium calculation principles, including the variance principle, which states the risk load should be proportional to the variance of X_k, i.e.:

$$R_k = \kappa \mathbb{V}\mathrm{ar}[X_k], \tag{1}$$

where κ is a constant. For more on premium calculation principles see, for example, Bühlmann (1970), Gerber (1979) or Goovaerts et al. (1984).

When expenses can be categorized as fixed or variable, the traditional actuarial approach is to determine the gross premium according to the equation

$$G = \frac{\mathbb{E}[X_k] + e_\mathrm{F}}{1 - e_\mathrm{V} - e_\mathrm{R}},$$

where e_F is the fixed expense per policy, e_V is the variable expense factor and e_R is the combined profit and risk factors (expressed as a percentage of the premium); see, for example, McClenahan (1996) or Booth et al. (1999, Chapter 14).

The population of potential insureds is often not homogeneous, i.e., they have different expected costs and other risk characteristics. In such cases, actuarial classification criteria are used to place the insureds into relatively homogeneous groups that sufficiently large to yield reliable cost estimates. Each group is then priced separately. See, for example, Finger (1996) for more on risk classification.

1.2. Life insurance

1.2.1. Individual life insurance

For individual life insurance, the biggest potential problem faced by insurers is adverse selection, i.e., persons in relatively poor health seeking to purchase life insurance at favorable rates.[1] To reduce the possibility of adverse selection, insurers subject potential insureds to a family health background check and may require a medical exam, the extent of which depends on the amount of insurance involved. The results of the medical exam may determine whether the individual is denied insurance or offered insurance for an extra (higher) premium; or (iii) offered the desired amount of insurance as a standard risk or as a preferred risk at the best (select) rate.

The results of the medical underwriting helps the insurer to determine the current state of the individual's health and ascertain the individual's expected future mortality. This allows the insurer to choose the appropriate age and/or

[1] The problem of adverse selection in life insurance has been noted and dealt with as a practical problem by actuaries for over 100 years. In life insurance, the primary method used to combat adverse selection is the medical exam. Jureidini and White (2000) provide an excellent review of the history of medical exams for life insurance purposes. Since the seminal paper by Akerlof (1970), however, the problem of adverse selection in insurance has been studied almost exclusively by economists and insurance academics. Many of these authors have focused on the problem of the existence and nature of a competitive equilibrium. Important contributions to the solution of this problem are due to Rothschild and Stiglitz (1976), Wilson (1977) and Spence (1978). Dionne and Harrington (1992) contain a reproduction of some of the important papers published on adverse selection in an insurance context. Macho-Stadler and Pérez-Castrillo (2001, Chapter 4) treat adverse selection as a special case of the principal-agent problem.