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Insurance: Mathematics and Economics 36 (2005) 93–101
INSURANCE MATHEMATICS & ECONOMICS
www.elsevier.com/locate/econbase

# Degree of downside risk aversion and self-protection

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Received May 2004; received in revised form October 2004; accepted 2 November 2004

**Abstract**

This paper shows that, identifying individuals with their utility functions, $-v'''(x)/v''(x) \leq -u'''(x)/u''(x)$ for all $x$ implies that individual $v$'s optimal choice of self-protection expenditure is larger than individual $u$'s, provided that marginal increases in self-protection expenditure from $u$'s optimal choice are mean-preserving. The result clarifies the relationship between self-protection and downside risk aversion and underscores the interpretation of $-u'''(x)/u''(x)$ as a measure of the strength of $u$'s downside risk aversion relative to his own risk aversion because a mean-preserving increase in self-protection expenditure is shown to effect a special combination of a downside risk increase and a mean-preserving contraction.


*JEL classification:* D81

IE30:

*Keywords:* Self-protection; Risk-aversion; Downside risk-aversion; Prudence measure

## 1. Introduction

In their pioneering work, Ehrlich and Becker (1972) define self-protection to be the expenditure on reducing the probability of suffering a loss[2] and highlight its conceptual distinction from self-insurance (the expenditure on reducing the severity of loss). In particular, unlike self-insurance, self-protection may be attractive to both risk averters and risk lovers, and market insurance and self-protection can be complements. Dionne and Eeckhoudt (1985) further show that a more risk-averse individual does not always purchase more self-protection. Briys and

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[1] I received valuable comments on an earlier version of the paper from Louis Eechoudt, Carmen Menezes, Harris Schlesinger, and X. Henry Wang. Very helpful and encouraging comments from an anonymous referee are also gratefully acknowledged. The usual disclaimer applies.
[2] Examples of self-protection includes crime prevention measures such as the purchase of burglary alarms, paying a higher price for a safer car or healthier food or a house in a less crime-prone area, the purchase of fire prevention equipments such as smoke detectors, etc. The problem of self-protection is also embedded in the usual moral hazard models and in models of environmental protection.

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doi:10.1016/j.insmatheco.2004.10.005

Schlesinger (1990) explain this phenomenon by showing that self-protection in general does not reduce the riskiness of individuals' final wealth and instead it may result in an increase in downside risk as is defined by Menezes et al. (1980). Menezes et al. (1980) also show that an individual is averse to an increase in downside risk if and only if his von Neumann-Morgenstern (VNM) utility function has a positive third derivative. In trying to clarify the relationship between an individual's degree of risk aversion and his optimal choice of self-protection, McGuire et al. (1991) and Jullien et al. (1999) show that if a less risk-averse individual's optimal choice of self-protection is such that the resulting loss probability is less than a critical "switching" level, then a more risk-averse individual's optimal choice will be higher than the less risk-averse. In a related endeavor Chiu (2000) shows that a risk-averse individual with a VNM utility function $u(x)$ is willing to pay more than the actuarially fair price (i.e., the expected reduction in loss) for a reduction in the probability of loss if the initial probability of loss is below a threshold and the threshold is determined by $-u'''(x)/u''(x)$, which is known as the prudence measure (Kimball, 1990) in the literature but is shown in a special context to have the interpretation of measuring the individual's downside risk aversion relative to his own risk aversion.

These earlier contributions thus illustrate the complexity of the effects of self-protection and our still inadequate understanding of them. While considerable efforts have been made to clarify the relationship between risk aversion and the choice of self-protection, Briys and Schlesinger (1990) and Chiu (2000) suggest that individuals' aversion to downside risk at least plays an equally important role in determining the optimal choice of self-protection. Building on these earlier studies, we show in this paper that, identifying individuals with their VNM utility functions, $-v'''(x)/v''(x) \leq -u'''(x)/u''(x)$ implies that individual $v$'s optimal choice of self-protection expenditure is larger than individual $u$'s, provided that marginal increases in self-protection expenditure from $u$'s optimal choice are mean-preserving or actuarially fair (i.e., the marginal expenditure is equal to the marginal reduction in the expected loss). The result clarifies the relationship between self-protection and downside risk aversion and underscores the interpretation of the prudence measure $-u'''(x)/u''(x)$ as a measure of one's downside risk aversion relative to one's own risk aversion in a static model: the effect of a mean-preserving increase in self-protection expenditure is also shown to be a special combination of a downside risk increase and a mean-preserving contraction (MPC) (i.e., a reduction in riskiness in the sense defined by Rothschild and Stiglitz (1970)) satisfying the conditions for $-u'''(x)/u''(x)$ to measure $u$'s strength of downside risk aversion relative to his own risk aversion. An individual whose aversion to downside risk is weaker relative to his preference for MPCs will thus opt for such an increase in self-protection expenditure.

The rest of the paper is organized as follows. Section 2 sets out the basic model and the main analytical result. Section 3 develops the interpretation of the main result in relation to a measure of an individual's downside risk aversion relative to his risk aversion. Section 4 concludes.

## 2. The basic model and result

We consider individuals with initial wealth $w$ who are at risk of losing $l$ with probability $p$, where $l < w$. Individuals $u$'s and $v$'s preferences are represented by the thrice-differentiable von Neumann-Morgenstern utility $u(\,)$ and $v(\,)$ respectively, where $u'(\,) > 0$ and $v'(\,) > 0$. Individuals can choose to pay $e$ to reduce the probability of loss by $\epsilon(e)$. We assume $\epsilon(0) = 0$. Individual $u$'s problem of choosing $e$ to maximize his expected utility is thus:

$$\max_{0 \leq e \leq e^p} U(e) \equiv (p - \epsilon(e))u(w - l - e) + (1 - p + \epsilon(e))u(w - e),$$

where $e^p$ is defined by $\epsilon(e^p) = p$. ($v$'s optimization problem is analogously given.) We assume that the functions $\epsilon(\,)$, $u(\,)$, and $v(\,)$ are such that the second-order conditions are satisfied and the solutions to the optimization problems are unique and internal. $u$'s and $v$'s optimal choices of self-protection, $e_U^*$ and $e_V^*$, are therefore given respectively