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Axiom of solvency and portfolio immunization under random interest rates

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Abstract

The paper concerns the interest rate risk management of insurance companies. It is assumed that assets and liabilities are stochastic processes and that regulatory demands on solvency are satisfied under the basic TSIR. Lower bounds on change in the portfolio present value are established when the interest rates change randomly. The bounds may be used to immunize the portfolio valuation when arbitrary changes of the interest rate scenarios are considered. An application to construct an optimal funding method for defined benefit pension plans is discussed in detail. In particular, a theoretical background is provided for the following conservative long-term strategy for the interest rate risk management: (1) assume a pessimistic (i.e. relatively low) interest rate scenario to discount cash flows of both assets and liabilities; (2) under this term structure of interest rates arrange the accumulated net assets cash flow to be the smallest concave majorant of accumulated liability cash flow. It is shown that this strategy is quite likely to give a positive change in the portfolio surplus in response to changing the future interest rate scenario. What is more, it leads to the least possible decrease of the portfolio net worth in the least favorable circumstances. A further improvement of this strategy is proposed.

JEL classification: G11; G22; G23

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Long term asset-liability management essentially is focused on immunizing the portfolio value against the interest rate risk (C3 risk). F. Redington published his paper on immunization in 1952, and since then multitude of articles on this topic has appeared. An excellent review on the history of the idea of immunization, as well as on a recent research, can be found in Panjer (1998). From the very beginning, immunization was thought as a way to stabilize the balance sheet of an insurance company, but this fact has had a very limited influence on the conclusions so far. In particular, to the best of our knowledge, there are no research works investigating portfolio immunization

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under solvency requirements imposed on the insurers by regulatory bodies. The aim of this paper is to revise the immunization methodology in order to work with this concrete problem.

Let us consider a stream of non-negative liability outflows $\{L_1, \dots, L_n\}$, due at dates $\{1, \dots, n\}$ and a stream of non-negative net asset inflows $\{A_1, \dots, A_n\}$, occurring at the same dates. From a mathematical point of view, it is of minor importance if the costs and return on capital are added to liabilities or subtracted from assets; for the sake of simplicity we subtract from assets. Net assets are resulting from that operation. We allow that some of assets or liabilities be equal to zero by the investment horizon n. We assume throughout the paper that the assets and liabilities are valued by an actuary using the same discount function corresponding to a given basic term structure of interest rates (TSIR). Namely, let P_k denote the present value at time $t = 0$ of one monetary unit due at $t = k$; then $a_k = P_k A_k$ and $l_k = P_k L_k$ denote the present values of A_k and L_k at $t = 0$, respectively. Clearly, the present value of the whole portfolio of assets and liabilities, V, is given by

$$V = \sum_{k=1}^{n} a_k - \sum_{k=1}^{n} l_k.$$

Throughout the paper V is treated as a target (net) value of the portfolio under the basic TSIR. It is reasonable that the regulatory bodies require that $V \geq 0$; however the immunization bounds given in the paper are valid also in the case $V < 0$. The problem we are interested in is how the target value of the portfolio can change when the basic TSIR changes. Let V' denote the present values of the whole portfolio when the basic TSIR is perturbed. Similarly, let $a'_k = P'_k A_k$ and $l'_k = P'_k L_k$ denote the present values of the asset A_k and the liability L_k at time $t = 0$, respectively, under the new TSIR. Then

$$V' - V = \sum_{k=1}^{n} (a_k - l_k) \left(\frac{P'_k}{P_k} - 1 \right). \tag{1}$$

The function $f(k) = P'_k / P_k$ is usually called shift factor. Redington (1952) formulated the immunization problem as follows: find conditions under which $V' - V \geq 0$ for any change of TSIR. However, the immunization postulate that $V' - V \geq 0$ for any scenario of interest rates is contradictory (except for some special circumstances) with the postulate that the market does not admit arbitrage opportunity (see, e.g. Panjer, 1998). Therefore, $V' - V$ may be non-negative only for some special types of change of TSIR. For example, Hürlimann (2002), Montrucchio and Peccati (1991), and Uberti (1997) considered immunization of the portfolio against perturbations of the basic TSIR with convex or convex-like shift factors. On the other hand, Fong and Vasicek (1984) considered arbitrary interest rates scenarios and have obtained a lower bound on the change in the end-of-horizon value of the portfolio. Such lower bound is the product of two terms and may be negative for some perturbations. The first term is a function of the change of TSIR only, while the second one depends solely on the structure of the investment portfolio. All the above mentioned results concern deterministic interest rates, assets and liabilities.

In contrast with the above authors, we start analyzing the immunization problem with an observation that any insurance regulatory institution will not allow to operate an insurance company with a negative net worth in a considered time period. It means that the basic TSIR used by an actuary in a balance sheet to value assets and liabilities must also imply that $V \geq 0$ and the following inequalities hold

$$\sum_{s=1}^{t} a_s \geq \sum_{s=1}^{t} l_s \quad \text{for } t = 1, \dots, (n-1). \tag{2}$$

Let us observe that

$$S_t = \sum_{s=1}^{t} a_s - \sum_{s=1}^{t} l_s$$