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Insurance: Mathematics and Economics 36 (2005) 399–420

INSURANCE MATHEMATICS & ECONOMICS
www.elsevier.com/locate/ime

A numerical method to find the probability of ultimate ruin in the classical risk model with stochastic return on investments

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Received July 2004 ; received in revised form February 2005; accepted 11 March 2005

Abstract

Let $\psi(y)$ be the probability of ultimate ruin in the classical risk process compounded by a linear Brownian motion. Here y is the initial capital. We give sufficient conditions for the survival probability function $\phi = 1 - \psi$ to be four times continuously differentiable, which in particular implies that ϕ is the solution of a second order integro-differential equation. Transforming this equation into an ordinary Volterra integral equation of the second kind, we analyze properties of its numerical solution when basically the block-by-block method in conjunction with Simpsons rule is used. Finally, several numerical examples show that the method works very well.

Keywords: Ruin probability; Volterra equation; Block-by-block method

1. Introduction

The problem of finding the probability of ultimate ruin for a risk process has been the subject of a vast number of papers in the insurance literature. By far the majority of these papers are concentrated on the analytical aspects of the problem, but there is also a quite considerable number that deal with numerical methods to actually calculate this evasive probability. These papers are roughly divided into two groups, the largest attempting to solve the relevant integral- or integro-differential equation numerically, and the other using Monte Carlo techniques. Examples of papers in this latter group are Lehtonen and Nyrhinen (1992) and Asmussen and Binswanger (1997) for the classical risk process while Asmussen and Nielsen (1995) allows for a deterministic return on investments. However, it is

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0167-6687/$ – see front matter
doi:10.1016/j.insmatheco.2005.02.008

clear from the paper by Paulsen and Rasmussen (2003) that successful use of Monte Carlo techniques in the presence of a stochastic return on investments is by no means a routine task.

When it comes to numerical methods for the classical risk process, the literature abounds with suggestions, see e.g. De Vylder (1996) for an overview of some of these methods. Surprisingly few of the suggested methods take advantage of the vast knowledge about integral equations in the numerical literature, and as a consequence many do not pay much attention to the error rate inherent in any numerical method. An exception is the paper by Ramsay and Usabel (1997) where the method of product integration is used to solve the Volterra integral equation for the ruin probability.

It should also be mentioned that there is a considerable literature dealing with numerical methods for calculating the ruin probability in finite time, see De Vylder (1996) for several of these. More recent papers allowing for a constant return on investments are Dickson and Waters (1999), Brekelmans and De Waegenaere (2001), and Cardoso and Waters (2003). See also the survey paper by Paulsen (1998a). Since very little is known analytically here, numerical methods are even more important than in the infinite time case.

In this paper we will be concerned with infinite time ruin probabilities for the diffusion perturbated classical risk process compounded by a linear Brownian motion, see Section 2 for the details. When there is no Brownian motion, the abovementioned paper by Dickson and Waters (1999) presents several numerical methods from the literature and show by numerical examples that some of them work well. However, none of these methods belong to the standard numerical methods developed by numerical analysts to solve the relevant equations, and consequently Dickson and Waters do not discuss the error rate of the various methods. In Section 2, we use integration by parts to represent the survival probability as a linear Volterra integral equation of the second kind, and then use (in one case a modification of) the standard block-by-block method in conjunction with the Simpson rule to solve this equation. Partly relying on known results in the numerical literature, we prove in Section 3 that the numerical solution has an error of order 4. For this to hold it is necessary that the survival function is four times continuously differentiable, and some effort is spent in Section 2 to prove this fact. We also prove in Section 3 that the error rate of order 4 still holds when functions appearing in the kernel of the integral operator are calculated numerically using the Simpson rule. Finally, in Section 4, several numerical examples are given showing that the method really works. Here it is also shown how known asymptotic results can be used to approximate very small ruin probabilities and also how they can be used to facilitate calculations when ruin probabilities are very heavy tailed.

In this paper we concentrate on the case when assets earn return on investments. Needless to say, our methods would of course work equally well for the simpler case with no such return.

2. The model and some theoretical results

We will assume that all processes and random variables are defined on a filtered probability space $(\Omega, \mathcal{F}, \mathbf{F}, P)$ satisfying the usual conditions, i.e. $\mathcal{F}_t$ is right continuous and P-complete.

The basic process is the surplus generating process

$$P_t = pt + \sigma_P W_{P,t} - \sum_{i=1}^{N_t} S_i, \quad t \geq 0. \tag{2.1}$$

Here W_P is a standard Brownian motion independent of the compound Poisson process $\sum_{i=1}^{N_\cdot} S_i$. We denote the intensity of N by λ, and let F be the distribution function of the S_i. It is assumed that F is continuous and concentrated on $(0, \infty)$. In the literature P is called a classical risk process perturbated by a diffusion.

The interpretation of (2.1) is that p is the premium intensity, N_P is the claim number process and the $\{S_{P,i}\}$ are the claim sizes. The assumption $F_P(0) = 0$ assures that they are positive. The Brownian term $\sigma_P W_P$ is meant to take care of small perturbations in the other two terms.