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Generating schemes for long memory processes: regimes, aggregation and linearity ☆

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Abstract

This paper analyses a class of nonlinear time series models exhibiting long memory. These processes exhibit short memory fluctuations around a local mean (regime) which switches randomly such that the durations of the regimes follow a power law. We show that if a large number of independent copies of such a process are aggregated, the resulting processes are Gaussian, have a linear representation, and converge after normalisation to fractional Brownian motion. Alternatively, an aggregation scheme with Gaussian common components can yield the same result. However, a non-aggregated regime process is shown to converge to a Levy motion with infinite variance, suitably normalised, emphasising the fact that time aggregation alone fails to yield a FCLT. Two cases arise, a stationary case in which the partial sums of the process converge, and a nonstationary case in which the process itself converges, the Hurst coefficient falling in the ranges $(\frac{1}{2}, 1)$ and $(0, \frac{1}{2})$, respectively. We comment on the relevance of our results to the interpretation of the long memory phenomenon, and also report

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some simulations aimed to throw light on the problem of discriminating between the models in practice.

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1. Introduction

Autoregressive unit roots are a popular feature of econometric models, not least thanks to the attractive feature that stationarity can be induced by either differencing or forming cointegrating linear combinations of economic time series. However, an often remarked drawback with this approach is that many important series do not seem to fall, logically or empirically, into either of the $I(0)$ (stationary) or $I(1)$ (difference stationary) categories. Their movements may appear mean reverting, for example, yet too persistent to be explained by a stationary, short-memory process. The fractionally integrated class of long memory models provide a seemingly attractive alternative, in which the $I(1)/I(0)$ dichotomy is replaced by a continuum of persistence properties. In this class, a time series x_t has the representation $(1-L)^d x_t = u_t$ for $-\frac{1}{2} < d < \frac{1}{2}$ where u_t is a stationary, weakly dependent, zero mean process. See Granger and Joyeux (1980), Hosking (1981) and Beran (1994) among other well-known references on these models. As detailed in Davidson (2002b), cointegration theory can be adapted straightforwardly to this set-up. Davidson and de Jong (2000) show that the normalised partial sums of such series converge to fractional Brownian motion (fBM) under quite general conditions.

However, this approach has its own drawback, that fractional integration cannot be modelled by difference equations of finite order. Thinking of a time series model as describing a representative agent's actions, incorporating hypothesised behavioural features such as adjustment lags and rational expectations, it is natural to see this behaviour as conditioned on the 'recent past', represented by at most a finite number of autoregressive lags. Unless a unit root is involved, all such models exhibit exponentially short memory. It is impossible to generate hyperbolic memory decay from finite order difference equations. Long memory models necessarily involve the infinite history of the observed process, and devising economic models with this structure is, for obvious reasons, a lot harder than constructing finite order models. A series can, of course, be modelled to have long memory characteristics through an error correction model driven by exogenous long memory; but finding a plausible route to endogenous long memory is difficult.

The attempts to devise such mechanisms in the literature have abandoned the representative-agent dynamic framework in favour of some form of cross-sectional aggregation. Since macroeconomic time series are not in fact generated by the behaviour of a fictional representative agent, but represent the net effect of many heterogeneous agents interacting, cross-sectional aggregation is a plausible