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On leverage in a stochastic volatility model

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Abstract

This paper is concerned with the specification for modelling financial leverage effect in the context of stochastic volatility (SV) models. Two alternative specifications co-exist in the literature. One is the Euler approximation to the well-known continuous time SV model with leverage effect and the other is the discrete time SV model of Jacquier et al. (J. Econometrics 122 (2004) 185). Using a Gaussian nonlinear state space form with uncorrelated measurement and transition errors, I show that it is easy to interpret the leverage effect in the conventional model whereas it is not clear how to obtain and interpret the leverage effect in the model of Jacquier et al. Empirical comparisons of these two models via Bayesian Markov chain Monte Carlo (MCMC) methods further reveal that the specification of Jacquier et al. is inferior. Simulation experiments are conducted to study the sampling properties of Bayes MCMC for the conventional model.

JEL classification: C11; C15; G12

Keywords: Bayes factors; Leverage effect; Markov chain Monte Carlo; Nonlinear state space models; Quasi maximum likelihood; Particle filter

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1. Introduction

Stochastic volatility (SV) models have gained much attention in both the option pricing and financial econometrics literature (see Ghysels et al. (1996) and Shephard (1996) for reviews of SV models and their applications). For example, Melino and Turnbull (1990) show that prices of European call options on currencies based on the basic SV models are more accurate than those based on the Black–Scholes model. Kim et al. (1998) provide evidence of better in-sample-fit of the basic SV model relative to GARCH-type models. Despite these documented advantages, it is known that the basic SV model can be too restrictive for many financial time series.

An important and well documented empirical feature in many financial time series is the financial leverage effect (Black, 1976; Christie, 1982; Engle and Ng, 1993). When such an asymmetric feature is not permitted in the SV model, option prices could be substantially biased (Hull and White, 1987). Motivated by this empirical evidence, Harvey and Shephard (1996) propose a SV model with leverage effect which is termed the asymmetric SV (ASV1 hereafter) model. This model is the Euler approximation to the continuous time asymmetric SV model widely used in the option price literature; see for example Hull and White (1987), Wiggins (1987), and Chesney and Scott (1989). Harvey and Shephard fit the model to stock data using a quasi-maximum likelihood (QML) method while Meyer and Yu (2000) fit it to an exchange rate series using a Bayesian Markov chain Monte Carlo (MCMC) method. The findings of both papers reveal overwhelming evidence of the leverage effect. Motivated by the same empirical evidence, Jacquier et al. (2004) generalized the basic SV model by incorporating an asymmetric feature which is also termed the leverage effect (ASV2 hereafter). A Bayesian MCMC approach was then developed to estimate the ASV2 model and strong evidence of "leverage effect" was found in most financial time series considered. Chan et al. (2004) extend the specification of Jacquier et al. to a multivariate setting. Unfortunately, these two specifications are not identical although both are claimed to be able to capture the leverage effect. They differ in how the correlation of two error processes is modelled.

The main purpose of this paper is to compare these two alternative specifications. The results obtained in the present paper show that the ASV2 model is inferior to the ASV1 model, judged from both theoretical and empirical view points. Firstly, the ASV2 model is not consistent with the efficient market hypothesis because the model is not a martingale difference sequence. Secondly, while the interpretation of the leverage effect using a parameter in the ASV1 model is clear, the strict interpretation of leverage is not obvious in the ASV2 model. Finally, I find the ASV2 model to be empirically inferior to the ASV1 model when S&P500 and Center for Research in Security Prices (CRSP) data are used.

To relate both SV models to the financial leverage effect, I derive a Gaussian nonlinear state space representation for each model. I then fit them to two stock indices using a Bayesian MCMC method. The choice of the MCMC method for inferences is mainly due to a result obtained by Andersen et al. (1999)