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Quasi-maximum likelihood estimation for conditional quantiles

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Abstract

In this paper, we construct a new class of estimators for conditional quantiles in possibly misspecified nonlinear models with time series data. Proposed estimators belong to the family of quasi-maximum likelihood estimators (QMLEs) and are based on a new family of densities which we call 'tick-exponential'. A well-known member of the tick-exponential family is the asymmetric Laplace density, and the corresponding QMLE reduces to the Koenker and Bassett's (Econometrica 46 (1978) 33) nonlinear quantile regression estimator. We derive primitive conditions under which the tick-exponential QMLEs are consistent and asymptotically normally distributed with an asymptotic covariance matrix that accounts for possible conditional quantile model misspecification and which can be consistently estimated by using the tick-exponential scores and Hessian matrix. Despite its non-differentiability, the tick-exponential quasi-likelihood is easy to maximize by using a 'minimax' representation not seen in the earlier work on conditional quantile estimation.

JEL classification: C22; C51; C61

Keywords: Tick-exponential family; Minimax representation; QMLE; Conditional quantiles; Asymptotic distribution; Misspecification

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1. Introduction

A vast majority of the empirical work in economics and finance has traditionally focused on models for conditional means. Over the last decade, however, applied literature has devoted increasing attention to other aspects of conditional distributions, such as their quantiles (see, e.g., Koenker and Hallock, 2000). This important empirical work has in turn rekindled the interest of the academic community in the theoretical problem of conditional quantile estimation and inference, a problem which we address in this paper.

Since the seminal work by Koenker and Bassett (1978), numerous authors have used a quantile regression framework for conditional quantile estimation (see, e.g., Koenker and Bassett, 1982; Powell, 1986; Portnoy, 1991; Koenker and Zhao, 1996; Kim and White, 2003) and specification testing (see, e.g., Koenker and Bassett, 1982; Zheng, 1998; Bierens and Ginther, 2001; Horowitz and Spokoiny, 2002; Koenker and Xiao, 2002; Kim and White, 2003). Common finding of the extant literature is that the quantile regression estimator has nice asymptotic properties under various data dependence structures and for a wide variety of conditional quantile models. While many efforts have been made in generalizing the existing results to new models and data structures, surprisingly little attention has been devoted to finding alternative semi-parametric estimators for conditional quantiles. There are yet important theoretical and practical benefits in having different conditional quantile estimators available. For example, it is difficult, if not impossible, to address questions such as finding a minimal covariance estimator or constructing a Hausman-type model specification test, if we only have one conditional quantile estimator available.

In contrast to the prior literature, our approach to conditional quantile estimation is based on a quasi-maximum likelihood. As already demonstrated in the context of conditional mean estimation, the quasi-maximum likelihood framework allows one to simply determine the class of all consistent estimators (see, e.g., Gourieroux, Monfort and Trognon, 1984; Bollerslev and Wooldridge, 1992; White, 1994). It is a well known result that there exists a variety of non-Gaussian quasi-maximum likelihood estimators (QMLEs) which, under standard regularity conditions and provided that they belong a linear-exponential family, are consistent for the parameters of a correctly specified conditional mean model.[1] In this paper, we derive an analog result valid in the context of conditional quantile estimation. In other words, we show that there exists an entire class of QMLEs—class that we call 'tick-exponential'—which is consistent for the parameters of a correctly specified model of a given conditional quantile. In the particular case where the tick-exponential density equals an asymmetric Laplace (or double exponential) density, the tick-exponential QMLE reduces to the standard Koenker and Bassett (1978) quantile regression estimator.

The QMLE generalization is not only of theoretical interest but also has a substantial practical contribution. For example, it provides an alternative approach to the conditional quantile covariance matrix estimation. While in the Gaussian

[1]Examples of non-Gaussian quasi-likelihoods used in the empirical work include: Gamma, Bernoulli, Poisson and Student-t (see, e.g., Bollerslev, 1987).