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ELSEVIER Journal of Economic Dynamics & Control 29 (2005) 1385–1425
JOURNAL OF Economic Dynamics & Control
www.elsevier.com/locate/econbase

Second-, third-, and higher-order consumption functions: a precautionary tale

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Received 20 November 2003; accepted 13 August 2004
Available online 5 November 2004

Abstract

No analytic solution exists to the consumption problem of an agent who faces uninsurable income shocks, though perturbation methods can be used to derive approximate solutions. This paper shows it is straightforward to extend the well-known second-order consumption function to third order. However, for every n there is a threshold interest rate below which the nth-order correction diverges. This puts a bound on the accuracy that can be achieved with perturbation methods. Even a second-order consumption function can perform spectacularly worse than a zeroth-order function that disregards precautionary saving, and this cannot be rectified by going to higher orders.

JEL classification: C63; E21

Keywords: Perturbative methods; Consumption function; Precautionary saving; Skewness

1. Introduction

In recent years, macroeconomists have sought to refine their understanding of consumption and saving by augmenting the basic model underlying the

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0165-1889/$ - see front matter
doi:10.1016/j.jedc.2004.08.005

lifecycle-permanent income hypothesis with uninsurable, idiosyncratic sources of risk that lead agents to engage in precautionary saving. Unfortunately, when agents have constant relative risk aversion (CRRA) preferences, there is no analytic expression for consumption and saving functions in such an augmented model. Several researchers (a partial list would include Carroll (1997, 2001a), Leland (1968), Letendre and Smith (2001), Ludvigson and Paxson (2001), Skinner (1988), and Viceira (2001)) have, therefore, turned to using perturbation methods to investigate this consumption/saving model.[1] The present paper takes a detailed look at the application of perturbation methods in the context of a partial-equilibrium consumption/saving model where risk-free bonds are the only intertemporal asset.

While the terminology of perturbation theory is fairly new to the economics literature, the technology has been in use for as long as economics has been a mathematical science. Perturbative methods simply involve the approximation of a function by its Taylor expansion. Any linearization procedure is an example of perturbation theory in its most elementary form. More generally, these methods are an integral component of the macroeconomist's toolkit. They are used regularly to express endogenous variables as approximate functions of exogenous variables. Although perturbative methods have been pitched under this name most prominently as an adjunct to numerical methods (Judd, 1999), they actually produce an analytic result – albeit only an approximate result – with the power to reveal functional dependences that can only be inferred when numerical values are plugged into exogenous parameters from the start.

Perturbation theory expresses endogenous variables as power series in a dimensionless parameter, the perturbation parameter, and provides a prescription for computing each coefficient of the series in terms of lower-order coefficients. For small values of the perturbation parameter, the power series can then be approximated by the sum of a finite number of terms. Since this power series will converge for any value of the perturbation parameter below some radius of convergence, in principle one should be able to achieve any degree of accuracy as long as the perturbation parameter is below this radius. In the present context, however, perturbation methods are limited in their utility because their proper application depends on assumptions that do not hold here. In the infinite horizon, the power series for consumption is only defined up to a finite number of terms, so the view that a first- or second-order calculation is an initial step in a sequence of calculations which will ultimately converge to an exact answer is overly optimistic.

The starting point for this analysis is a paper by Skinner (1988), which evaluated the consumption function to second order with respect to the coefficient of variation of the income process for a finite-horizon model with income and interest-rate shocks.[2] This captures the lowest-order effects of the variance and precautionary

[1] Alternatively, one can use the intrinsically numerical approach of computing consumption functions either through value-function or Euler-equation iteration (Deaton, 1991). Instead of perturbing around the known solution to a solvable problem, this involves computing the relevant function on a finite set of points and then using interpolation or projection methods to evaluate the function at other points.

[2] Talmain (1998) has extended this result for a general utility function in the special case where the interest rate equals the discount rate.