Available online at www.sciencedirect.com

ScienceDirect

Journal of Economic Theory 158 (2015) 165–185

JOURNAL OF Economic Theory

www.elsevier.com/locate/jet

Maximal manipulation of envy-free solutions in economies with indivisible goods and money [☆]

Yuji Fujinaka [a,*], Takuma Wakayama [b]

[a] *Faculty of Economics, Osaka University of Economics, 2-2-8, Osumi, Higashiyodogawa-ku, Osaka, 533-8533, Japan*
[b] *Faculty of Economics, Ryukoku University, 67 Tsukamoto-cho, Fukakusa, Fushimi-ku, Kyoto 612-8577, Japan*

Received 6 March 2015; accepted 29 March 2015

Available online 7 April 2015

Abstract

We consider the problem of the fair allocation of indivisible goods and money with non-quasi-linear preferences. The purpose of the present study is to examine strategic manipulation under *envy-free* solutions. We show that under a certain domain-richness condition, each individual obtains the welfare level of his "optimal" *envy-free* allocation by maximally manipulating the solutions. This maximal manipulation theorem is helpful in analyzing the set of Nash equilibrium allocations in the direct revelation games associated with a given *envy-free* solution: if an *envy-free* solution satisfies a mild condition, the set of Nash equilibrium allocations in its associated direct revelation game coincides with that of *envy-free* allocations.

JEL classification: C72; C78; D63; D71

Keywords: Envy-freeness; Indivisible good; Manipulation; Nash implementation

[☆] The paper was previously entitled "Maximal manipulation in fair allocation." Tommy Andersson, Lars Ehlers, and Lars-Gunnar Svensson independently obtain results that are closely related to our results. We thank them for sending us the paper and communication. We are grateful to an associate editor, an anonymous referee, H. Reiju Mihara, Shuhei Morimoto, Tokinao Otaka, Toyotaka Sakai, and Rodrigo Velez for their valuable comments and suggestions. We also thank the participants of the seminar at Tokyo Metropolitan University, Kobe University, Hitotsubashi University, Tokyo Institute of Technology, and Summer Workshop on Economic Theory in 2010 (Otaru University of Commerce). Fujinaka gratefully acknowledges the financial support by KAKENHI (21830090). Wakayama gratefully acknowledges the financial support by KAKENHI (22730165, 22330061, 25380244).

* Corresponding author.
E-mail addresses: fujinaka@osaka-ue.ac.jp (Y. Fujinaka), wakayama@econ.ryukoku.ac.jp (T. Wakayama).
URLs: http://www.geocities.jp/yuji_fujinaka/ (Y. Fujinaka), http://www.geocities.jp/takuma_wakayama/ (T. Wakayama).

http://dx.doi.org/10.1016/j.jet.2015.03.014

1. Introduction

We consider the problem of the fair allocation of indivisible goods, or "objects," among a group of individuals when monetary transfers are allowed. We assume that each individual receives one and only one object and that the budget is balanced. Consider, for example, the following situation: when a group of friends rents a house, they must determine the room assignments and fairly divide the rent.[1] An "allocation" consists of an assignment of objects and a distribution of money.

One of the fundamental requirements for fairness in this problem is *envy-freeness*, which states that each individual prefers his own consumption bundle over that of any other individual. A number of studies have focused on *envy-freeness* because it has desirable properties: the existence of *envy-free* allocations is guaranteed under certain assumptions, and each *envy-free* allocation is *efficient* (Svensson, 1983; Alkan et al., 1991). On the other hand, it is well known that *envy-freeness* is incompatible with *strategy-proofness* (Alkan et al., 1991; Tadenuma and Thomson, 1995). This negative result, however, only means that at some preference profiles, telling the truth is not optimal for everyone; thus, we do not know what each individual can achieve through the manipulation of *envy-free* solutions.[2]

The purpose of our study is to identify the welfare that each individual can achieve by manipulating an *envy-free* solution. An allocation is defined as *i-optimal envy-free* if it is an *envy-free* allocation that is the most preferred by individual i among all *envy-free* allocations. We establish that given a preference profile, individual i can, through the optimal manipulation of a given *envy-free* solution, obtain a welfare level that is equivalent to that of the *i-optimal envy-free* allocation at that preference profile. This "maximal manipulation theorem" holds for every *envy-free* solution. That is, no matter which *envy-free* solution is employed, each individual can obtain the same level of welfare through his optimal manipulation. Moreover, our maximal manipulation theorem does not depend on quasi-linearity of preferences. To clarify this point, we introduce a domain-richness condition that allows individuals to have non-quasi-linear preferences. Our maximal manipulation theorem holds true whenever the preference domain is rich.

The maximal manipulation theorem helps us analyze the set of Nash equilibrium allocations in the direct revelation game associated with all *envy-free* solutions. By invoking this theorem, we show that for each preference profile, each Nash equilibrium allocation is *envy-free*, and, conversely, under a mild condition for solutions, called *extended non-discrimination*, each *envy-free* allocation is a Nash equilibrium allocation. That is, if an *envy-free* solution satisfies *extended non-discrimination*, then for each preference profile, the set of Nash equilibrium allocations in the associated direct revelation game coincides with the set of *envy-free* allocations at that profile. Furthermore, it turns out that on important preference domains, *extended non-discrimination* is necessary and sufficient for the coincidence between the set of Nash equilibrium allocations in the direct revelation game associated with *envy-free* solutions and the set of *envy-free* allocations.

The remainder of the present paper is organized as follows. We discuss related literature in the next section. Section 3 describes the model and introduces *envy-freeness* and *i-optimal envy-freeness*. We present the maximal manipulation theorem in Section 4. In Section 5, we apply the maximal manipulation theorem to examine equilibrium allocations in the manipulation games. We conclude in Section 6. A number of proofs are presented in Appendix A.

[1] This problem is referred to as the "room assignment-rent division problem" (e.g., Abdulkadiroğlu et al., 2004).

[2] An *envy-free* solution is a function that associates an *envy-free* allocation with each preference profile.