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A duality approach to continuous-time contracting problems with limited commitment ☆

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Abstract

We propose a duality approach to solving contracting models with either one-sided or two-sided limited commitment in continuous time. We establish weak and strong duality theorems and provide a dynamic programming characterization of the dual problem. The dual problem gives a linear Hamilton–Jacobi–Bellman equation with a known state space subject to free-boundary conditions, making analysis much more tractable than the primal problem. We provide two explicitly solved examples of a consumption insurance problem. We characterize the optimal consumption allocation in terms of the marginal utility ratio. We find that neither autarky nor full risk sharing can be an optimal contract with two-sided limited commitment, unlike in discrete-time models. We also derive an explicit solution for the unique long-run stationary distribution of consumption relative to income.

JEL classification: C61; D86; D91; E21

Keywords: Continuous-time contracts; Limited commitment; Risk sharing; Duality; Dynamic programming; Regulated Brownian motion

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1. Introduction

Many empirical studies find that idiosyncratic variation in consumption is systematically related to idiosyncratic variation in income, rejecting the hypothesis of full risk sharing (e.g., Cochrane, 1991; Mace, 1991, and Townsend, 1994). Instead of assuming exogenous market incompleteness, one important approach to reconciling this empirical evidence is to assume that individuals have limited commitment (e.g., Kocherlakota, 1996; Alvarez and Jermann, 2000, and Ligon et al., 2002). This assumption is motivated by the fact that debt repayments are costly to enforce. Debt collection, litigation, and income garnishment are costly, and the debtor may default on debt. In this case, individual income risks are only incompletely shared.

Although discrete-time dynamic models with limited commitment have been widely applied in economics and finance,[1] these models are typically difficult to solve analytically and numerical solutions are needed. The main contribution of this paper is to propose a duality approach in a continuous-time setup, which permits analytical solutions. We consider a consumption insurance problem between a principal and an agent analogous to the problems analyzed by Thomas and Worrall (1988), Kocherlakota (1996), Alvarez and Jermann (2000), Ligon et al. (2002), and Ljungqvist and Sargent (2004).[2] The continuous-time setup is analytically convenient and allows us to derive sharp and transparent results. We find that the usual dynamic programming approach using the agent's continuation value as a state variable in the primal problem delivers a nonlinear Hamilton–Jacobi–Bellman (HJB) equation with state constraints. The state space of the continuation value is endogenous in models with two-sided limited commitment. Such a nonlinear HJB equation typically does not admit any analytical solution and is difficult to analyze even numerically. By contrast, the dual problem transforms the primal problem with participation constraints into an unconstrained problem, which delivers a linear HJB equation subject to free-boundary conditions. Technically, it is an instantaneous (or singular) control problem, similar to those analyzed in Harrison and Taksar (1983), Harrison (1985), and Stokey (2008).

We study the link between the dual and primal problems and establish the weak and strong duality theorems. We provide a dynamic programming characterization of the dual problem using the usual state variables (individual incomes) together with additional costate variables. The costate variables are the cumulative amounts of the Lagrange multipliers associated with the intertemporal participation constraints, starting from pre-specified initial conditions. These costate variables are nonnegative and increasing processes. They are also the control variables in the dual problem and rise whenever the participation constraints bind.

In the case of one-sided limited commitment, there is only one costate variable, which is associated with the agent's participation constraints. To facilitate discussion, we first consider the case in which the principal and the agent have an identical discount rate. In this case, the costate variable is also equal to the ratio of the marginal utilities of the principal and the agent. The agent's current income and the marginal utility ratio constitute the state variables of the

[1] Other examples include applications to wage contracts by Thomas and Worrall (1988), sovereign debt by Bulow and Rogoff (1989), Kletzer and Wright (2000), and Hellwig and Lorenzoni (2009), asset markets by Kehoe and Levine (1993) and Alvarez and Jermann (2000, 2001), optimal taxation by Chari and Kehoe (1993), business cycles by Cooley et al. (2004), international business cycles by Kehoe and Perri (2002), consumption inequality by Krueger and Uhlig (2006) and Krueger and Perri (2006), the welfare effects of a progressive tax by Krueger and Perri (2011), political economy by Acemoglu et al. (2011), and asset bubbles by Kocherlakota (2008) and Miao and Wang (2011, 2012).

[2] The principal and the agent can be interpreted in different ways in different contexts. They can be two households, a planner and a household, or a firm and a worker.