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Strategic stability in Poisson games

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Abstract

In Poisson games, an extension of perfect equilibrium based on perturbations of the strategy space does not guarantee that players use admissible actions. This observation suggests that such a class of perturbations is not the correct one. We characterize the right space of perturbations to define perfect equilibrium in Poisson games. Furthermore, we use such a space to define the corresponding strategically stable sets of equilibria. We show that they satisfy existence, admissibility, and robustness against iterated deletion of dominated strategies and inferior replies.

JEL classification: C63; C70; C72

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1. Introduction

Poisson games (Myerson [29]) belong to the broader class of games with population uncertainty (Myerson [29], Milchtaich [28]). Not only have these games been used to model voting behavior but also more general economic environments (see, e.g., Satterthwaite and Shneyerov [34], Makris [22,23], Ritzberger [33], McLennan [24], Jehiel and Lamy [17]). In these

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models, every player is unaware about the exact number of other players in the population. Each player in the game, however, has probabilistic information about it and, given some beliefs about how the members of such a population behave, can compute the expected payoff that results from each of her available choices. Hence, a Nash equilibrium in this context is a description of behavior for the entire population that is consistent with the players' utility maximizing actions given that they use such a description to form their beliefs about the population's expected behavior.

Similarly to standard normal form and extensive form games, one can easily construct examples of Poisson games where not every Nash equilibrium is a plausible description of rational behavior. In particular, Nash equilibria in Poisson games can be in dominated strategies. Indeed, many applications of Poisson games (see, e.g., Myerson [30], Maniquet and Morelli [21], Bouton and Castanheira [6], Núñez [31]; among others) focus on undominated strategies in their analysis. In addition, there are also examples in the applied literature of Poisson games that use some other kind of refinements (Hughes [16], Bouton [5], Bouton and Gratton [7]). Hence, it seems worthwhile exploring, also in games with population uncertainty, what can be said from a theoretical standpoint about which Nash equilibria are the most reasonable and to propose a definition that selects such equilibria for us.

Following the main literature on equilibrium refinements, we start focusing our attention on *admissibility*. That is, the principle prescribing players not to play dominated strategies (Luce and Raiffa [20, p. 287, Axiom 5]). Furthermore, as in Kohlberg and Mertens [19], we also require that the solution be robust against iterated deletion of dominated actions. Unfortunately, as it is already well known, such an iterative process can lead to different answers depending on which order is chosen to eliminate the dominated strategies. The response to this caveat is defining a set-valued solution concept and requiring that every solution to a Poisson game contain a solution to any game that can be obtained by eliminating dominated strategies. Of course, a definition of such a concept for Poisson games should be guided by the literature on Strategic Stability for finite games (Kohlberg and Mertens [19], Mertens [26,27], Hillas [14], Govindan and Wilson [13]). In broad terms, a strategically stable set is a subset of Nash equilibria that is robust against every element in some given space of perturbations. The choice of such a space determines the properties that the final concept satisfies and the perturbations are just a means of obtaining the game theoretical properties that we desire (Kohlberg and Mertens [19, p. 1005, footnote 3]). As argued above, a strategically stable set of equilibria should only contain undominated strategies. Furthermore, it should always contain a strategically stable set of any game obtained by eliminating a dominated strategy. However, De Sinopoli and Pimienta [10] show that the main instrument used to define strategic stability in normal form games—i.e. Nash equilibria of strategy perturbed games—fails to guarantee that players only use undominated strategies when applied to Poisson games.

Thus, before defining strategically stable sets of equilibria in Poisson games we need to find the appropriate space of perturbations that guarantees that every member of the stable set is undominated. It turns out that the "right" space of perturbations is of the same nature as the one used in infinite normal-form games (Simon and Stinchcombe [36], Al-Najjar [1], Carbonell-Nicolau [9]) and different from the one used in finite games (Selten [35]) even if players have finite action sets. Once this class of perturbations has been identified, it can be reinterpreted as a collection of perturbations of the best response correspondence. Then, a stable set is defined as a minimal subset of fixed points of the best response correspondence with the property that every correspondence that can be obtained using such perturbations has a fixed point close to it.

As an illustration of stable sets in Poisson games we construct a referendum game with a threshold for implementing a new policy (see Example 5). In this example, every voter prefers