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Robustness of public equilibria in repeated games with private monitoring ☆

Richard McLean [a], Ichiro Obara [b,*], Andrew Postlewaite [c]

[a] *Rutgers University, United States*
[b] *University of California, Los Angeles, United States*
[c] *University of Pennsylvania, United States*

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Abstract

A repeated game with private monitoring is "close" to a repeated game with public monitoring (or perfect monitoring) when (i) the expected payoff structures are close and (ii) the informational structures are close in the sense that private signals in the private monitoring game can be aggregated by some public coordination device to generate a public signal whose distribution is close to the distribution of the public signal in the public monitoring game. We provide a sufficient condition for the set of uniformly strict perfect public equilibria for a public monitoring game to be robust in nearby private monitoring games in the sense that they remain equilibria with respect to the public signal that is generated by such public coordination devices with truthful reporting. Our sufficient condition requires that every player is informationally small in a well-defined sense.

JEL classification: C72; C73; D82

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* Corresponding author.

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1. Introduction

Cooperation within groups is an important and commonly observed social phenomenon, but the way in which cooperation arises is one of the least understood questions in economics. The theory of repeated games has improved our understanding by showing how coordinated threats to punish can prevent deviations from cooperative behavior, but much of the work in repeated games rests on very restrictive assumptions that all players share the same public information either perfectly or imperfectly. For the case in which each player can observe all other players' actions directly (perfect monitoring), Aumann and Shapley [5] and Rubinstein [34] proved a folk theorem without discounting, and Fudenberg and Maskin [13] proved a folk theorem with discounting. For the case in which each player observes a noisy public signal (imperfect public monitoring), Abreu, Pearce and Stacchetti [1] characterized the set of pure strategy sequential equilibrium payoffs and Fudenberg, Levine and Maskin [14] proved a folk theorem.

But a theory that rests on the assumption that there is common knowledge of a sufficient statistic about all past behavior is, at best, incomplete. Such a theory is of little help in understanding behavior in groups in which there are idiosyncratic errors in individuals' observations of outcomes.[1] For many problems, it is more realistic to consider players who possess only partial information about the environment and, most importantly, players may not know the information possessed by other players. In such problems, players may communicate their partial information to other players in order to build a "consensus" regarding the current situation, which can be used to coordinate their future behavior. In this view, repeated games with public information can be thought of as a reduced form of a more complex interaction involving information sharing.

This point of view leads us to examine the *robustness* of equilibria with public monitoring when monitoring is private, but "close" to public monitoring. For example, one can think of a situation in which information contained in the public signal is dispersed among the players in the form of noisy private signals. If the amount of information contained in each player's private signal is negligible, then it is tempting to consider the game with such private signals and the underlying game with public signals as being "close." In this paper, we examine whether an equilibrium with public monitoring remains an equilibrium with respect to a public signal generated from private monitoring and communication, and whether (and how) players can be induced to reveal their private information.

To make these ideas precise, consider a public monitoring game (G, π) and a private monitoring game (G', p), where G and G' are normal form games with public monitoring and private monitoring respectively. In (G, π), each action profile a generates a public signal y from a set Y with probability $\pi(y|a)$. In (G', p), each action profile a generates a private signal profile $s = (s_1, .., s_n)$ with probability $p(s|a)$. In our analysis of the private monitoring game (G', p), we will augment the model with a "public coordination device" ϕ that chooses a public coordinating signal (possibly randomly) from Y based on the reported profile of private signals. In this expanded game, players choose an action profile a, observe their private signals $(s_1, .., s_n)$, and publicly announce the (not necessarily honest) profile $(s'_1, .., s'_n)$. A public coordinating signal $y \in Y$ is then selected with probability $\phi(y|s'_1, .., s'_n)$. If the players report their private signals truthfully, then the probability that the realized public coordinating signal is y given a and ϕ is equal to $p^{\phi}(y|a) = \sum_{s \in S} \phi(y|s)p(s|a)$. We say that (G, π) and (G', p) are close when G and G' are close in terms of expected payoffs and there exists a public coordinating device ϕ such

[1] For example, team production in which each individual observes the outcome with error lies outside this framework.