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Pairwise comparison dynamics for games with continuous strategy space

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Abstract

This paper studies pairwise comparison dynamics for population games with continuous strategy space. We show that the pairwise comparison dynamic is well-defined if certain mild Lipschitz continuity conditions are satisfied. We establish Nash stationarity and positive correlation for pairwise comparison dynamics. Finally, we prove global convergence and local stability under general deterministic evolutionary dynamics in potential games, and global asymptotic stability under pairwise comparison dynamics in contractive games.

JEL classification: C72; C73

Keywords: Evolutionary dynamics; Population games; Continuous strategy space

1. Introduction

Over the past few decades, evolutionary game theory has been an active area of research in economics, biology, computer science, and sociology, and other fields that study interactions among large numbers of participants. However, most studies are restricted to settings in which the strategy space is finite. This restriction limits the use of population games in applications whose strategy spaces are naturally modeled as continuous, including games of timing, effort choice games, bargaining games, and oligopoly games, among others.

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The games mentioned above have the same property that agents' actions are chosen from some intervals of real numbers. Another kind of games that involve continuous strategy sets is incomplete information games with a continuum of types, for example, auctions. This paper provides some new insights into an increased scope of the use of population games in applications with continuous strategy settings.[1]

When the strategy set is finite, a population state can be described by a real-valued vector with dimension equal to the cardinality of the strategy set. For the case of continuous strategy space, a population state is described by a probability measure over the strategy space. This introduces technical challenges.

For population games with continuous strategy sets, Bomze [2,3] defines the replicator dynamic in the Banach space of finite signed measures with the variational norm. Bomze [3] shows that the replicator dynamic is well-defined if certain Lipschitz continuity conditions are satisfied for the mean payoff function. Oechssler and Riedel [16,17] follow this line of research on the replicator dynamic. They show that Bomze's conditions are always satisfied in pairwise encounters if the underlying pairwise payoff function is bounded, and give a more fruitful result on evolutionary stability.

Evolutionary dynamics describe the aggregate consequences of individual agents employing simple myopic rules to decide how to act. Different rules lead to different dynamics. The dynamic that attracts most people to study and is used most often in applications is the replicator dynamic, due to its origin from biology. The replicator dynamic is *imitative* in the sense that, under such dynamic, when an agent receives an opportunity to switch strategies, he chooses a candidate strategy at random according to the population state, i.e., according to how popular that candidate strategy is.

This paper introduces pairwise comparison dynamics for games with continuous strategy space. Unlike the replicator dynamic, pairwise comparison dynamics are *direct*[2] in the sense that a revising agent chooses a candidate strategy at random according to a fixed reference measure; in particular, a strategy's popularity does not influence the probability with which it is chosen as a candidate strategy.[3] Under pairwise comparison dynamics, the revising agent switches to the candidate strategy at a positive rate if and only if its payoff is higher than his current strategy's payoff. In the special case of the Smith [27] dynamic, the rate is proportional to the difference between the candidate strategy's payoff and the payoff of the agent's current strategy.

In the present paper, we provide a general framework to derive the *mean dynamic* for population games in continuous strategy settings. The framework and derivation are in the same spirit as those using *revision protocols* by Sandholm [21,23,24] in the finite strategy case. Basically, revision protocols are the rules that individual agents follow to switch strategies. From the mean dynamic, not only pairwise comparison dynamics but also other deterministic evolutionary dynamics (such as the replicator dynamic, the BNN dynamic, and logit dynamics) can be derived.[4]

[1] For some examples of the application of evolutionary dynamics in continuous strategy games, see Friedman and Ostrov [7], Hofbauer, Oechssler and Riedel [9], Hu [11], Lahkar and Riedel [12], Louge and Riedel [13], and Oechssler and Riedel [16,17].

[2] Cf. Sandholm [23], Section 4.3.2.

[3] While the replicator dynamic is not direct but imitative (since the reference measure is not fixed but taken to be the current population state; see Remarks 1 and 2 in Section 2 below), the replicator dynamic is considered in the general framework and analysis of the present paper (e.g., the global convergence and local stability results for potential games apply to the replicator dynamic; see Remarks 4 and 5 in Section 5 below).

[4] See Remark 1 in Section 2 below.