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Notes

Reputation in the presence of noisy exogenous learning ☆

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Abstract

This note extends Wiseman [6] to more general reputation games with exogenous learning. Using Gossner's [4] relative entropy method, we provide an explicit lower bound on all Nash equilibrium payoffs of the long-lived player. The lower bound shows that when the exogenous signals are sufficiently noisy and the long-lived player is patient, he can be assured of a payoff strictly higher than his minmax payoff.

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Keywords: Reputation; Repeated games; Learning; Relative entropy

1. Introduction

Wiseman [6] studies an infinitely repeated chain store reputation game in which the short-lived entrants receive noisy exogenous signals about the type of the long-lived incumbent. He shows that a sufficiently patient long-lived incumbent can effectively build a reputation and assure himself of a payoff strictly higher than his minmax payoff provided the exogenous signals are noisy enough.

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This note extends Wiseman [6] to more general reputation models with exogenous learning. The analysis is built on Gossner [4] who introduces the relative entropy approach to the study of standard reputation models in Fudenberg and Levine [3] and obtains an explicit lower bound on all equilibrium payoffs. This paper shows Gossner's [4] powerful tool can also be naturally adapted to reputation models with exogenous learning.[1] We provide an explicit lower bound on all Nash equilibrium payoffs to the long-lived player which is a modification of that in Gossner [4]. The lower bound is characterized by the commitment action, discount factor, prior belief and in particular how noisy the learning process is. In general this lower bound is lower than that in Gossner [4] because of learning and these two bounds coincide if the exogenous signals are completely uninformative.

The rest of the paper is organized as follows. In Section 2, we describe the reputation model with exogenous learning. Section 3 presents the main result. Section 4 applies the obtained lower bound to the example considered in Wiseman [6] and discusses the result. All the proofs are in Appendix A.

2. Model

2.1. Reputation game with exogenous learning

We consider the canonical reputation model (Mailath and Samuelson [5], Chapter 15) in which a fixed stage game is infinitely repeated. The stage game is a two-player simultaneous-move finite game of private monitoring. Denote by A_i the finite set of actions for player i in the stage game. Actions in the stage game are imperfectly observed. At the end of each period, player i only observes a private signal z_i drawn from a finite set Z_i. If an action profile $a \in A_1 \times A_2 \equiv A$ is chosen, the signal vector $z \equiv (z_1, z_2) \in Z_1 \times Z_2 \equiv Z$ is realized according to the distribution $\pi(\cdot|a) \in \Delta(Z)$.[2] The marginal distribution of player i's private signals over Z_i is denoted by $\pi_i(\cdot|a)$. Both $\pi(\cdot|a)$ and $\pi_i(\cdot|a)$ have obvious extensions $\pi(\cdot|\alpha)$ and $\pi_i(\cdot|\alpha)$ respectively to mixed action profiles. Player i's ex-post stage game payoff from his action a_i and private signal z_i is given by $u_i^*(a_i, z_i)$. Player i's ex ante stage game payoff from action profile $(a_i, a_{-i}) \in A$ is $u_i(a_i, a_{-i}) = \sum_{z_i} \pi_i(z_i|a_i, a_{-i}) u_i^*(a_i, z_i)$. Player 1 is a long-lived player with discount factor $\delta \in (0, 1)$ while player 2 is a sequence of short-lived players each of whom only lives for one period. In any period t, the long-lived player 1 observes both his own previous actions and private signals, but the current generation of the short-lived player 2 only observes previous private signals of his predecessors.

There is uncertainty about the type of player 1. Let $\Xi \equiv \{\xi_0\} \cup \hat{\Xi}$ be the set of all possible types of player 1. ξ_0 is the *normal type* of player 1. His payoff in the repeated game is the average discounted sum of stage game payoffs $(1 - \delta) \sum_{t \geq 0} \delta^t u_1(a^t)$. Each $\xi(\hat{\alpha}_1) \in \hat{\Xi}$ denotes a *simple commitment type* who plays the stage game (mixed) action $\hat{\alpha}_1 \in \Delta(A_1)$ in every period independent of histories. Assume $\hat{\Xi}$ is either finite or countable. The type of player 1 is unknown to player 2. Let $\mu \in \Delta(\Xi)$ be player 2's prior belief about player 1's type, with full support.

At period $t = -1$, nature selects a type $\xi \in \Xi$ of player 1 according to the initial distribution μ. Player 2 does not observe the type of player 1. However, we assume that the uninformed player 2 has access to an exogenous channel which gradually reveals the true type of player 1.

[1] Ekmekci et al. [2] also applies the relative entropy approach to the reputation game in which the type of the long-lived player is governed by an underlying stochastic process.

[2] For a finite set X, $\Delta(X)$ denotes the set of all probability distributions over X.