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Notes

Rental harmony with roommates ☆

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Abstract

We prove existence of envy-free allocations in markets with heterogenous indivisible goods and money, when a given quantity is supplied from each of the goods and agents have unit demands. We depart from most of the previous literature by allowing agents' preferences over the goods to depend on the entire vector of prices. We then show how our theorem may be applied in two related problems: Existence of envy-free allocations in a version of the cake-cutting problem, and existence of equilibrium in an exchange economy with indivisible goods and money. Our proof uses Shapley's K-K-M-S theorem and Hall's marriage lemma.

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1. Introduction

A central concept in the literature on economic fairness is *envy-freeness* [7,24,16] – an allocation is envy-free if no agent prefers the share allocated to another agent over his own share. In this note we study existence of envy-free allocations when the goods to be allocated are indivisible and heterogenous, and when in addition there is one perfectly divisible good (e.g., money).

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We assume that each agent has a demand for only one of the indivisible goods and that there is a given quantity supplied of each good.

While there are many real-life examples that can fit into this framework, we will use for concreteness the terminology of room-assignment and rent-division: Several rooms with different characteristics and given capacities are available in a house, and the total rent for the house needs to be divided between the rooms. In this context, envy-freeness boils down to a market clearing condition: A price is assigned to each room such that when each agent chooses his favorite room (given the prices) supply exactly equals demand and the market clears. Following Su [21], we call such a situation *rental harmony*. Note that even though we use the terms 'rooms' and 'capacities', we do not make the assumption that the agents to whom a given good is allocated, whom we call *roommates*, receive a joint ownership of the same physical object. Rather, a room with capacity 7 stands for an indivisible good of which 7 units are supplied, and the roommates represent the 7 agents who received these units. When we say that the price of the room is p we mean that each unit costs $p/7$.

There is quite a vast literature dealing with different aspects of this model. Some of the earlier works include [2,15,22,23]. Where we depart from most of the previous works (with a couple of exceptions – see below) is in the type of preferences that agents may have. Namely, it has been assumed in earlier works that each agent's preferences are defined over room-price pairs, i.e., if r, r' are two rooms with prices p and p' respectively, then each agent can say whether he prefers to get room r at price p or room r' at price p'. In our model an agent's favorite room may be a function of the *entire vector of prices*. Thus, asking whether an agent prefers (r, p) to (r', p') is not meaningful in our context, since the answer may depend on the prices of other rooms.

There are several reasons why this is an important generalization. First, there may be 'rational' reasons for agents' preferences over rooms to be affected by the entire vector of prices. This may be the case, for instance, if we view the choice of a room as only part of a larger 'consumption plan'. For a concrete example, assume that a forward looking agent needs to choose between three types of cars, say High (H), Intermediate (I) and Low (L), with corresponding prices $p_H > p_I > p_L$. If p_H is very high then an agent's preferred option may be to buy type I and hold it for a long period of time. But if p_H is reduced then the agent may prefer to buy L initially (saving a larger part of his budget) and upgrade to H later on when he has accumulated more wealth. Thus, his choice shifted from I to L even though the prices of these cars did not change.

Prices can also affect preferences if there is incomplete information about the quality of the rooms, in which case prices may serve as a signaling device. For instance, real-estate prices in two neighboring suburbs may provide information about their relative qualities. An increase (or decrease) in the price of houses in one of them may therefore affect the desirability of the other. Another reason for a similar effect is when agents take into account the fact that prices affect choices of other agents. In such an interactive situation there are plausible scenarios in which the entire vector of prices influences agents' optimal choices, for example if the price of a neighboring room indicates the identity of its future inhabitants.

Another reason to consider such general preferences is that framing effects and other well-documented 'behavioral biases' may be affecting choices in ways that the standard model cannot capture. For example, assume that rooms A and B have similar characteristics while room C is very different from the other two. Assume further that at a given price vector p with $p_A = p_B$ the agent's preferred choice is room C. If the price of A increases then room B may become more attractive as it offers similar value as room A for a 'bargain' price. The agent may then choose B instead of C, even though the prices of these rooms have not changed.