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On the space of players in idealized limit games ☆

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Abstract

This paper demonstrates *the* class of atomless spaces that accurately models the space of players in a large game which represents an idealized limit of a sequence of finite-player games. Through two examples, we show that arbitrary atomless probability spaces, in particular, the Lebesgue unit interval, may not be appropriate to model the space of players of an idealized limit. This inappropriateness hinges on the fact there is a convergent sequence of *exact* pure-strategy Nash equilibria in the sequence of finite-player games, while the idealized limit game of the sequence does *not* have any equilibrium. Instead, a saturated probability space is shown to be not only *sufficient* but also *necessary*, to model the space of players in any proper idealized limit. This complements the study of large games with a bio-social typology in Khan et al. [10] as such a connection between finite-limiting and idealized continuum-limit games was not able to be obtained in their framework.

JEL classification: C72; D50; C65

Keywords: Large games; Games with traits; Idealized limit; Saturated probability space; Pure-strategy Nash equilibrium; Weak closed-graph property

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1. Introduction

In many economic environments in society, an individual's action has a negligible effect on the aggregate summary of the actions of all other individuals. A continuum of agents, modelled by an atomless measure space, is assumed to be "the most natural model" (e.g. Aumann [3]) for capturing the numerical negligibility of an individual in an economy. An atomless probability space is now also standard for modelling the space of players in a societal context where each player is strategically negligible in the theory of large (non-cooperative) strategic games. By assuming that a player's payoff depends on a statistical summary of societal actions as well as on his or her individual action, the theory of large games is now well established.[1] The social identity literature emphasizes the importance of including the traits of players—be they biological or socio-economic—into the notion of player interdependence (e.g. Akerlof and Kranton [1]). To address this issue in large games, a comprehensive and analytically tractable framework has recently been offered in Khan et al. [10], in which the payoffs of players depend on their own actions and societal responses which include not only a societal action summary but also on an underlying summary of traits.

A large economy, with an atomless probability space to model the space of agents, is a good proxy to large finite economies.[2] In particular, a large economy can be treated as an idealized limit of a converging sequence of large finite economies.[3] Therefore, besides the existence of equilibria in a large economy itself, another important question to ask is whether a convergent sequence of equilibria in a convergent sequence of finite economies converges to an equilibrium in the idealized limit of the finite economies. Normally, a positive answer to such a question can be obtained under the general conditions of various environments—for example, pure exchange economies as in Hildenbrand and Mertens [7], strategic market games as in Dubey et al. [4], and large strategic games as in Green [5], among many others.

However, in general the answer is not always positive. In this note, we provide a set of counterexamples which challenge the claim, and its underling intuition, concerning the relation between finite-player environments and their idealized continuum limits. To be sure, Examples 1 and 2 in Khan et al. [10] have demonstrated a discrepancy between a continuum game and its finite analogues: the non-existence of equilibria in the continuum game and the existence of equilibria in its finite counterparts. But in their framework, the continuum game can not be the idealized limit of finite games.[4] Within a framework similar to that presented in Khan et al. [10], we present two examples that involve job-seeking opportunities in *two* cities: the game with the

[1] See the survey by Khan and Sun [11] on the existence of equilibria, and Jara-Moroni [8] on rationalizability in large games.

[2] See Anderson [2] for the approximate core equivalence result in large finite economies; also see Sun and Nicholas [15] on the study of compatibility of efficiency and incentives in the large economy and McLean and Postlewaite [13,14] for the approximated cases in large finite economies.

[3] We follow the convention and say that a sequence of finite economies converges to a large economy with a continuum of agents if the induced distributions of the players' characteristics in finite economies converge weakly to the induced distribution of characteristics in the limit game; e.g. Hildenbrand [6].

[4] As the convergence of games refers to the weak convergence of distributions of characteristics (e.g. Footnote 3), the convergence of games with different trait distributions *can not* even be asked meaningfully in the framework of Khan et al. [10]. More specifically, as a player's characteristics contain both trait and payoff, in order to have a convergent sequence of games, players' payoffs in all games in the sequence must share the same domain. And thus, all games must share the same distribution of traits as the distribution of traits is used to construct payoffs of a large game in Khan et al. [10]. Hence, when the limit game has an atomless distribution of traits, there *can not* exist any sequence of finite-player