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Notes

Non-existence of continuous choice *functions* ☆

Hiroki Nishimura, Efe A. Ok *

Department of Economics, New York University, 19th West 4th Street, New York, NY 10012, United States

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Abstract

Let X be a compact, or path-connected, metric space whose topological dimension is at least 2. We show that there does not exist a continuous choice function (i.e., single-valued choice correspondence) defined on the collection of all finite feasible sets in X. Not to be void of content, therefore, a revealed preference theory in the context of most infinite consumption spaces must either relinquish the fundamental continuity property or allow for *multi*-valued choice correspondences.
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1. Introduction

The primitives of revealed preference analysis for an individual are a universal set X (of choice alternatives), a collection $\mathcal{A}$ of nonempty subsets of X (to serve as the collection of all choice problems that the agent may potentially encounter), and a correspondence c mapping each element of $\mathcal{A}$ to a nonempty subset of that element (which is interpreted as the choice correspondence of the individual that tells us what she may choose in any given choice problem). In such a context, taking the choice correspondence c as single-valued (so that c is a *function* from

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* Corresponding author.
E-mail addresses: hiroki@nyu.edu (H. Nishimura), efe.ok@nyu.edu (E.A. Ok).

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$\mathcal{A}$ into X) often simplifies the analyses considerably. Especially in the recent body of research on boundedly rational choice theory, this modeling strategy is widely adopted.

It is, however, important to note that taking choice functions as primitives of analysis have behavioral implications. Indeed, positing that in every choice problem one can identify a unique alternative to choose assumes away some potentially interesting traits such as indifference and indecisiveness, thereby limiting the foundational nature of the involved model. (To wit, a preference relation over risky alternatives deduced from a choice *function* says that no nondegenerate lottery has a certainty equivalent.) This is, of course, not a novel point, but one that has been debated in the folklore extensively. In the present note, we would like to contribute to this debate by making a formal observation about single-valued choice correspondences.

In most economic frameworks, the universal sets of alternatives are infinite, and they are modeled as metric spaces. (In demand theory, for instance, this space is often taken as $\mathbb{R}^n_+$, in risk theory as the convex hull of a collection of points in a normed linear space, and in the theory of intertemporal choice as some suitable sequence space.) Revealed preference analyses in such contexts posit some form of continuity (such as the closed graph property) on the part of the choice correspondences. Not only is this reasonable, but it is often necessary for deriving utility functions, or at least for ensuring the existence of extrema with respect to preference (or other types of binary) relations induced from choice behavior. The goal of this note is to show that unless the alternative space has a rather esoteric structure, or it is topologically equivalent to an interval in the real line, no choice function can be continuous on a domain that contains all finite choice problems. For example, if X is any (nontrivial) normed linear space and $\mathcal{A}$ is the collection of all nonempty finite subsets of X, then there exists a continuous choice function on $\mathcal{A}$ (with $\mathcal{A}$ being metrized by the standard Hausdorff metric) if, and only if, X is homeomorphic to $\mathbb{R}$. In particular, there is no continuous choice function on the set of all nonempty finite subsets of $\mathbb{R}^n$ for any $n \geq 2$. In fact, so long as X is a metric space that contains a compact (or path-connected) subspace that is not homeomorphic to a subset of the real line, no choice function can be continuous on the set of all finite (or even doubleton) subsets of X.

It thus appears that there is an intrinsic clash between the properties of continuity and single-valuedness for choice correspondences. In idealized situations where one works with a complete set of single-observation choice data for finite sets, the data will often take a discontinuous form. In a nutshell, and in the words of one of the referees of this paper, "any (choice) theory that one comes up with to explain those data "exactly" (that is, without admitting that theory could also have generated a different set of single observations) will have to imply discontinuous behavior," unless the alternative space has a particularly simple structure.

In what follows, Section 2 introduces some preliminaries and Section 3 contains some examples, the statement of our main result and an application of it to the theory of incomplete preferences, as well as a brief discussion on the use of choice functions in revealed preference analysis. Section 4 provides a proof for our main result.

2. Preliminaries

2.1. Choice correspondences

Let X be a metric space. We denote the collection of all nonempty subsets of X that contain at most k elements by $\mathcal{F}_k(X)$. The collection of all nonempty finite subsets of X is then denoted by $\mathcal{F}(X)$, and that of all nonempty compact subsets of X by $\mathbf{k}(X)$. Throughout the present paper,

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