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Asymmetric all-pay auctions with interdependent valuations

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Abstract

I show that a unique equilibrium exists in an asymmetric two-player all-pay auction with a discrete signal structure, correlated signals, and interdependent valuations. The proof is constructive, and the construction can be implemented as a computer program and be used to derive comparative statics. I also characterize the set of equilibria when a reserve price is introduced.

JEL classification: D44; D82; D72

Keywords: Contests; All-pay; Auctions; Asymmetries; Interdependent valuations; Correlated signals

1. Introduction

This paper investigates a contest model in which two asymmetric contestants compete for a prize by expending resources. Each contestant has some private information that may affect both contestants' valuation for the prize, and the contestants are asymmetric in that their private information may be drawn from an asymmetric joint distribution and impact their valuations differently. For example, consider a research and development race in which the firm that develops the higher-quality product enjoys a dominant market position. Each firm may be partially informed about market demand, which determines the value of a dominant market position. Thus, firms' information may not be statistically independent. The value of a dominant market position

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may differ between the firms, because the profit associated with such a position may depend on firm-specific characteristics such as production costs and marketing expertise. Similar interdependencies and asymmetries in information and valuations for the prize arise in rent-seeking scenarios, such as lobbying, and in other competitions with sunk investments, such as competitions for promotions.

I model the contest as an asymmetric all-pay auction that allows for correlated signals and interdependent valuations. Each player privately observes a signal drawn from a finite ordered set, and these sets may differ between the players. Each player then places a bid, both players pay their bids, and the player with the higher bid wins the prize. The value of the prize is a player-specific function of both players' signals.

This formulation includes many natural settings as special cases. Singleton signal sets correspond to a complete-information all-pay auction (Hillman and Riley [7]). Valuations that depend only on a player's own signal correspond to a private-value setting. Identical valuations correspond to a common-value setting. A singleton signal set for only one player corresponds to a setting with one informed player and one uninformed player (with possibly different valuations).

The main result of the paper is a constructive characterization of the unique equilibrium. The equilibrium is constructed in a finite number of steps by proceeding from higher bids to lower bids. Each step identifies a pair of signals, one for each player, and an interval of bids on which the players bid when they observe these signals. The bidding densities in the interval are closely related to the equilibrium bidding densities in a complete-information all-pay auction with valuations that correspond to the observed signals. The construction ends when one player exhausts his bidding probability. Any remaining bidding probability of the other player is expended by bidding 0.

In many special cases the construction can be used to derive equilibrium properties of economic interest or characterize the equilibrium in closed-form. This can be used to derive comparative statics and answer various questions about the value of information. Section 5 enumerates several such cases. To the extent that players' valuations and the distribution of their signals are controlled by a designer, the results may be useful for designing contests that maximize effort, efficiency, or some other target function.

The model can also be used as a building block in a dynamic setting in which players make investments or choose how much information to acquire before the contest takes place. The effect of these strategic decisions can be modeled as determining the distribution of players' signals and how the signals affect players' valuations. The equilibrium characterization can then be used to solve for players' optimal behavior in the first stage.

I also consider the effect of a reserve price, which corresponds to a minimum investment necessary to win the contest. A player who bids below the reserve price loses, regardless of what the other player bids. I show that the structure of any equilibrium with a reserve price is closely related to that of the unique equilibrium without a reserve price. In particular, there exists a bid such that in any equilibrium with a reserve price, players' bidding behavior above the reserve price is a "right shift" of their bidding behavior above this bid in the contest without a reserve price. The introduction of a reserve price may lead to multiple equilibria, which only differ in the probabilities that players bid 0 and the reserve price. I characterize the set of equilibria, which are payoff equivalent.

Relation to the literature. The paper contributes to the large literature on perfectly-discriminating contests, in which there is a deterministic relationship between players' investments and