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Admissibility and assumption

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Abstract

Brandenburger et al. (2008) defined the epistemic condition *rationality and common assumption of rationality (RCAR)* in lexicographic type structures. We define RCAR in the space of coherent lexicographic belief hierarchies and show that it is an epistemic condition for iterated admissibility. We also show that some coherent lexicographic belief hierarchies cannot be represented as types in lexicographic type structures. An advantage of this approach is that RCAR is possible in a canonical hierarchy space.

JEL classification: C72; D80

Keywords: Lexicographic probability system; Higher-order beliefs; Coherence; Iterated admissibility; Epistemic game theory

1. Introduction

Iterated admissibility (IA) is a procedure for deleting inadmissible (i.e., weakly dominated) strategies from a game. In each iteration of the procedure, *all* inadmissible strategies of *all* players are deleted from the game. Applying IA to the strategic form of a game has flavors of both backward and forward induction, which are fundamental concepts in the extensive form analysis of games (see Brandenburger and Friedenberg, 2007).

Brandenburger et al. (2008, henceforth BFK) showed that *rationality and mth-order assumption of rationality (RmAR)* in complete lexicographic type structures is an epistemic condition for strategies that survive $m + 1$ rounds of the IA procedure. This solved an important puzzle

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that had challenged game theorists since Samuelson (1992) pointed out the inherent epistemic contradictions that arise in models that attempt to rationalize the procedure when all players have expected utility (EU) preferences.

Rationality and common assumption of rationality (RCAR) is the condition that RmAR holds for all m. A natural conjecture is that RCAR in complete lexicographic type structures is an epistemic condition for strategies that survive *all* rounds of the IA procedure. However, BFK proved that RCAR cannot be satisfied in any *continuous* complete lexicographic type structure, which seemingly put a damper on that possibility.[1] Keisler and Lee (2011) revived hope by showing that, for any given game,

(i) there exists some complete lexicographic type structure such that RCAR holds at some states in it; and
(ii) whenever RCAR holds at some state in a complete lexicographic type structure, RCAR epistemically characterizes the IA set of that game.

While these results were very encouraging, the complete lexicographic type structures constructed by Keisler and Lee (2011) depended on the given game. Keisler and Lee (2011) did not construct a *canonical* type structure in which RCAR is nonempty for all games. In this paper, we construct a canonical space of coherent lexicographic belief hierarchies and define the set of states in that space that satisfy RCAR, which corresponds to the following infinite list of statements.

(i) All players are rational.
(ii) (i) and all players assume (i).
(iii) (ii) and all players assume (ii).
... and so on.

Unlike type structures, which describe hierarchies implicity, this space describes hierarchies *explicitly*. We show that our approach is more general in the sense that some coherent hierarchies cannot be generated by any lexicographic type structure.[2] The presence of hierarchies that cannot be types is crucial to the intuition behind why RCAR is possible in the space of coherent belief hierarchies but not in continuous complete lexicographic type structures. The main result of this paper is that RCAR is a game-independent epistemic condition for IA strategies.

For these results, a naïve definition of coherent belief hierarchies will not suffice. The usual convention is to simply check the coherence of the *beliefs* that represent preferences. The more careful approach taken in this paper is to check the coherence of the *preferences* represented by the beliefs.[3] In that sense, it is related to the preference-based approach to beliefs about be-

[1] Faced with this discouraging result, alternate epistemic foundations for iterated admissibility were sought. Barelli and Galanis (2013) did so by introducing the notion of event-rationality and tie-breaking sets. Yang (2015) did so by weakening the assumption operator.

[2] It should be noted that these coherent hierarchies that cannot be types are different from the coherent hierarchies of *probabilistic* beliefs that cannot be types that arise in Heifetz and Samet (1999). The hierarchies that cannot be types in Heifetz and Samet (1999) arise due to the lack of restrictions on the σ-algebra on the underlying space of uncertainty. In our model, the underlying space of uncertainty is finite, which guarantees that all coherent hierarchies of *probabilistic* beliefs in our model can be types. The coherent hierarchies that cannot be types in our model are necessarily lexicographic.

[3] What we actually require is slightly stronger than this but nevertheless weaker than the naive approach to coherence.